AN EXPERIMENTAL STUDY OF BOUNDARY LAYER TRANSITION

Submitted by
CHARLES A. LEE
FLUID MECHANICS
LABORATORY SUPERINTENDENT

Prepared by
H. W. BENNETT

September, 1953

Prepared for
OFFICE OF NAVAL RESEARCH
Department of the Navy
WASHINGTON, D. C.

Under Office of Naval Research Contract Nonr-673(00)

KIMBERLY-CLARK CORPORATION
Research and Development Laboratories
NEENAH, WISCONSIN

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ABSTRACT

Transition on a smooth flat plate in a zero pressure gradient was studied in a wind-tunnel using conventional surface tube and hot wire techniques.

In the range of free stream turbulence used (0.10% to 0.5%) it is shown that the laminar oscillations predicted by the Tollmien-Schlichting theory of laminar layer stability play an important role in transition. The process of transition in this case is shown to be similar to the development of a turbulent wake from a vortex street, the amplified frequency feeding the rest of the spectrum. The effect of free-stream turbulence on the process was studied in terms of the energy concentrated at the amplified frequency. As anticipated, free stream turbulence hastens the feeding of energy from the amplified frequency to the rest of the spectrum. It is indicated that at a much higher level of free stream turbulence than used here, laminar oscillations will play no part in transition and possibly the phenomenon is controlled by the laminar separation theory proposed by Taylor.

INTRODUCTION

Since Prandtl introduced the concept of the boundary layer, a region near the surface of a body or the wall of a pipe where the velocity changes from that of the body right at the surface to the velocity of the free stream some distance away, much research has been accomplished to discover the characteristics of the layer, because of its importance in many fluid mechanics problems.

Consider a flat plate immersed lengthwise in a flowing stream. For some distance down the plate, if the leading edge is well sharpened, the plate is smooth, and the free stream is fairly non-turbulent, the boundary layer is laminar. The laminar boundary layer is thin and has the gradual velocity profile characteristic of laminar flow. After some distance, depending on the roughness of the plate, the free stream velocity, the free stream turbulence, etc., the boundary layer becomes turbulent and assumes the usual turbulent velocity profile. Although it is usually tacitly assumed that this transition occurs instantaneously, it is well known that this is not the case. Transition occurs over a considerable length of the plate and has its beginnings considerably upstream of the modification of the laminar velocity profile. It is toward a fuller understanding of this region, the conditions that determine where it will occur and the conditions within the region that the research discussed below is aimed.

This research, which was sponsored by the Office of Naval Research, was prompted by the need of fundamental information along this line

by those engaged in ship model testing, particularly at the David Taylor Model Basin. In model testing the frictional resistance cannot be scaled up directly because of the presence of a considerable relative length of laminar boundary layer on the model and a much shorter one on the prototype. The correction for this state of affairs has always been somewhat in doubt, in no small part due to a lack of knowledge concerning the position of transition on the model, and how this position is affected by the other variables in the water channel. One purpose of this work was to provide additional information on transition for this and other testing facilities.

There has been a considerable amount of research on this problem and it has been reviewed fairly extensively in some well known reports. Reference 1 gives a very comprehensive review of the literature and indicates a sizeable bibliography. For this reason, the literature review presented here will be fairly brief and will discuss mainly the previous work concerned with the effect of turbulence on transition on a smooth plate with zero pressure gradient.

SYMBOLS

x - Distance from leading edge of flat plate along the plate

y - Distance perpendicular from surface of plate

U_o - Mean velocity in free stream

U - Mean velocity at some point in boundary layer

u' - Root mean square value of velocity fluctuation in direction of flow

v', w' - Root mean square value of velocity fluctuation in y and z directions respectively

L - Scale of u' fluctuations

δ - Boundary layer displacement thickness

R_δ - Reynolds number based on δ and U_o

R_x - Reynolds number based on x and U_o

($R_\delta = 1.72\sqrt{R_x}$ for Blasius distribution)

β_r $= 2\pi f$

f = Frequency (c.p.s.)

q $= \frac{1}{2}\rho U_o^2$

ρ - Density

ν - Kinematic viscosity

E - Total energy in boundary layer

E_1 - Energy due to laminar oscillations

E_2 $= E - E_1$

$E_{1\ max}$ - Maximum height of discrete energy portion of energy spectrum

$E_{O\ max}$ - Maximum extrapolated energy at zero frequency through transition region a fixed distance above surface

CAUSES OF TRANSITION

The major items affecting transition on a flat plate are:

1. Roughness of the plate.
2. Free stream turbulence.
3. Pressure gradients along the plate.
4. Plate curvature.
5. Temperature variations.
6. Vibration, noise, etc.

In order to limit this study, only the effect of turbulence has been considered. For a discussion of some of the other effects, Ref. 1, 2, and 3 are offered.

EFFECT OF TURBULENCE

One of the earlier, more notable investigations of the effect of turbulence on transition was that of G. I. Taylor (Ref. 4). He theorized that transition was caused by separation, either momentary or permanent, in the laminar layer. Thus transition should be governed by the Karman-Pohlhausen parameter for laminar separation. For isotropic turbulence such as occurs in the wake of a grid, Taylor was able to relate the Karman-Pohlhausen parameter to the measurable free stream quantities, scale and intensity, and was thus able to relate the pressure forces causing intermittant separation to free stream quantities. Taylor evolved the relation $\frac{u'}{U_o} \frac{x}{L}^{1/5}$ as the parameter which determines at what R_x value the boundary layer will become turbulent. The above relation, of course, only applies to transition

in an isotropic turbulent field. Taylor confirmed that this parameter does control transition with fairly large values of free stream turbulence on flat plates, spheres, and elliptic cylinders. This theory has met with some objections in the last few years, not the least of which is that separation has not been proven to be a necessity for transition.

THEORY OF LAMINAR OSCILLATIONS

From a purely theoretical standpoint, it has been found that the laminar layer, by virtue of its thickness, the kinematic viscosity of the fluid involved, and the free stream velocity, is able to amplify certain frequencies of disturbance. This is in decided contrast to the above theory where the magnitude of the disturbance was felt to be the essential factor. Here the type of disturbance, particularly with regard to what frequencies are present, was considered to be the essential thing. This theory is discussed in some detail in Ref. 5. The essential result is the so called curve of neutral stability shown in Figure 1. Here $\frac{\beta_r \nu}{U_o^2}$ is plotted as a function of R_δ , where $\beta_r = 2\pi f$ (f is frequency), ν the kinematic viscosity, and U_o the free stream velocity.

This curve is a loop, the two branches meeting at infinity. This curve was first plotted by Schlichting and subsequently corrections have been made to the theory by Lin which changed somewhat the position of the right hand side of the loop (usually referred to as Branch II). Essentially, this loop defines which disturbances present

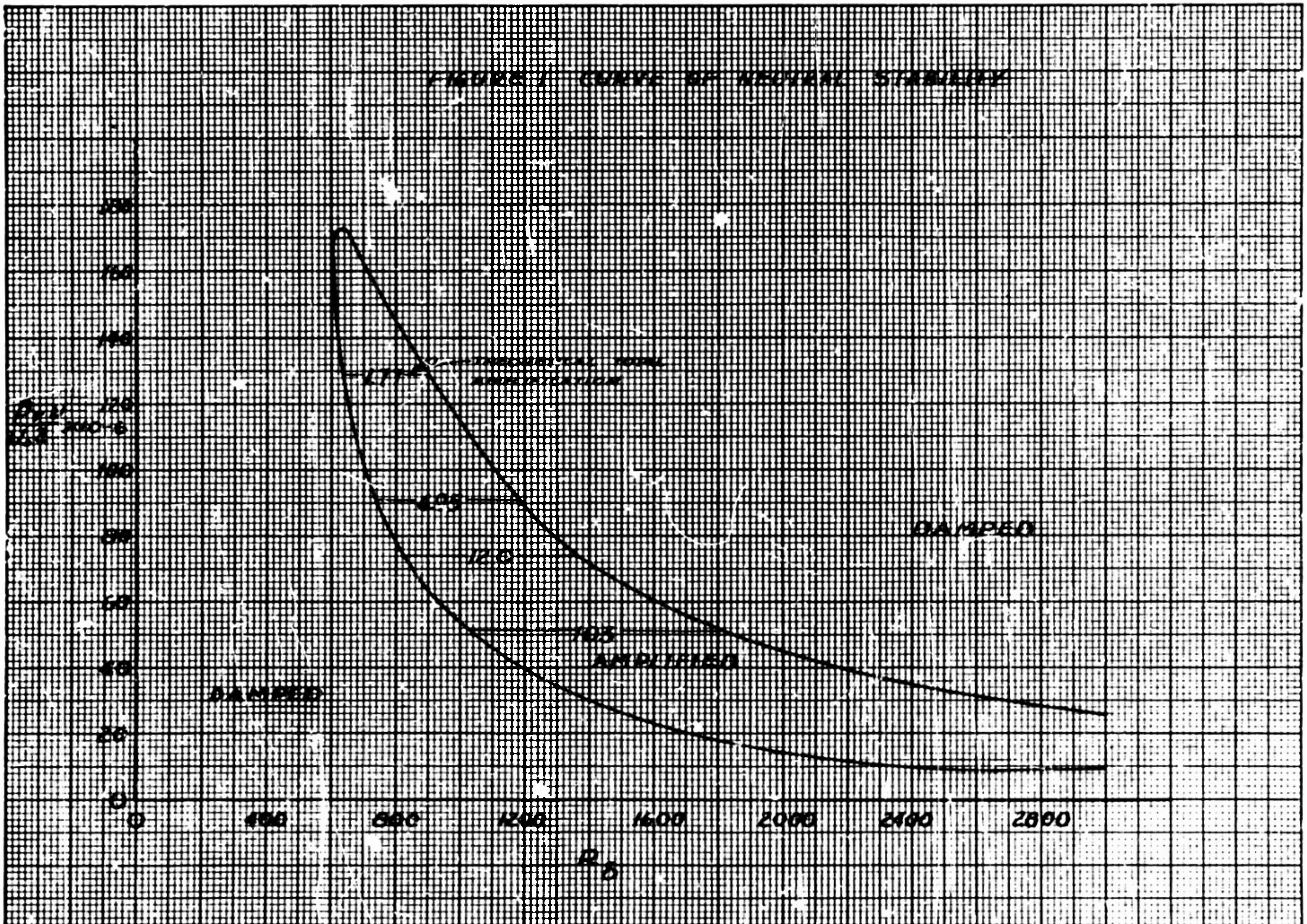


FIGURE 1 CURVE OF NEUTRAL STABILITY
DAMPED
AMPLIFIED
DAMPED
1.77
6.05
12.0
105
0
400
800
1200
1600
2000
2400
2800
20
40
60
80
100
120
140
160
180
200

in the boundary layer will be amplified and which disturbances will be damped. Any disturbance, free stream velocity and boundary layer Reynolds number combination which puts the situation at hand inside the loop, should theoretically cause the disturbance to be amplified. If the point falls outside the loop, the disturbance will be damped, and a point falling right on the curve should neither be damped nor amplified. Assuming a disturbance of constant frequency is impressed on the laminar layer at a small value of boundary layer Reynolds number, the disturbance will be damped until the boundary layer Reynolds number reaches the value where the frequency parameter intersects Branch I of the stability curve. From here on, as the point of observation is moved down the plate, the disturbance will be amplified until the boundary layer Reynolds number reaches the value of the intersection with Branch II. At this point the wave is again damped. The question remains, if a single frequequency is to be present in the boundary layer at a particular value of boundary layer Reynolds number, what should this frequency be? In Figure 1, the theoretical total amplification for several frequencies is shown. It is immediately obvious that at each particular value of R_{δ}, one particular frequency will have received more amplification than any other. This frequency, then, is the one which should predominate. If this is truly the case, as the Reynolds number is increased, the frequency which predominates should change. This will be discussed further in the light of the experimental results of this study.

These are some of the more important results of the theory. For a more complete discussion of both the theory and the results attained from it, Ref. 5 is offered.

For some time, this theory did not find favor, particularly among experimentalists, mainly because no one had been able to observe the oscillations which the theory predicted.

In 1938, however, Schubauer and Skramstad (Ref. 6) in a near classic investigation of transition with vanishing turbulence, were able to detect the astonishingly pure laminar oscillations. Evidently previous investigators had been unable to detect the oscillations because transition had occurred simultaneously or nearly simultaneously with the appearance of the waves.

Even more important than merely discovering the existance of the waves, Schubauer and Skramstad introduced oscillations of known frequency and amplitude into the boundary layer by vibrating a very thin metal ribbon at different frequencies and amplitudes in the boundary layer. By using a very narrow band-pass filter on the amplified output of a hot wire moved progressively down the plate, they were able to follow the growth and/or decay of the frequency being considered. Very conclusively they proved not only the existance of the waves, but also that the waves would grow and/or decay according to Figure 1. From this information, Schubauer and Skramstad formulated a new proposal of how transition occurs, particularly when the free stream turbulence is low.

They use the analogy of the formation of turbulence from a free vortex sheet. The boundary layer becomes wavelike, and eventually discrete eddies appear, as is usually assumed for a vortex sheet. These eddies in themselves are very unstable and eventually decay into turbulence. The decay into turbulence is gradual, the random turbulent motion only occurring a portion of the time. This explanation seems to fit the quantitative data presented by Schubauer and Skramstad better than the explanation of intermittant separation discussed above. In the oscillograms presented by Schubauer and Skramstad of hot wire output downstream of the vibrating metal ribbon, the first turbulent "bursts" observed in the laminar layer were not confined to the low velocity part of the cycle, i.e. if the turbulent "bursts" were truly caused by intermittant separation, one would expect that if vibrations were introduced into the laminar layer, the first signs of the randomness characteristic of turbulence would appear on the low side of the velocity cycle. This was not the case, which, to Schubauer and Skramstad, indicated that the transition by separation theory was open to some question, and that the idea of the boundary layer becoming wave-like, then breaking into discrete eddies which are very unstable, and soon decay into turbulent motion, fits the data somewhat more realistically.

Some support is lent to the theory of Schubauer and Skramstad by the work of Hama (Ref. 7). In connection with a study of the effect of a single roughness element on transition, Hama towed a flat plate horizontally in water and attached a small diameter pipe transversely to

the plate as a roughness element. Dye was injected through a small hole in the top of the pipe in the center of the plate into the surrounding water. He then photographed the motion of the dye as it progressed down the plate. At first vortices were emitted from the top of the pipe at a constant rate and would move down the plate with this orientation. Soon, however, neighboring vortices would catch up with one another forming a larger vortex. At some point further down the plate these vortices would "explode" and the motion became random. Hama postulates this might be transition.

Of course, conditions behind a single roughness element are not the same as conditions on a smooth plate, but the photographs of Hama do show that oriented vortices in the laminar boundary layer are not stable and seem to explode suddenly some distance down the plate. In the case of a smooth plate, the vortices are a product of the amplified boundary layer oscillations and in the wake of a roughness element the vortices are, of course, caused by the element. Nevertheless, the "exploding" of the waves is probably the same in both cases.

PROBABILITY TRANSITION THEORY

One further approach to the problem is that taken by Emmons (Ref. 8). In an observation on a water-table analogy to supersonic flow, Emmons observed boundary layer transition. It could be viewed clearly as the thin layer of water, when it became turbulent, was considerably changed in appearance with either transmitted or reflected light. Emmons noted that transition is not a clearly defined phenomenon, a

point which will be brought out further below, but is rather an intermittant process. He points out that by virtue of disturbances carried into the layer by outside turbulence, plate roughness, vibration, etc., the laminar layer is disturbed. When these disturbances reach a certain value, a turbulent "burst" occurs. This turbulent "spot", as Emmons calls them, moves along with the fluid and gradually fans out, making turbulent all before it. He points out that the "bursts" which are noted by the hot wire are these turbulent spots passing over the wire. The farther down the plate one observes the flow, the larger the number of turbulent spots which have been born upstream and hence the larger percent of the time the particular point under observation is turbulent. Emmons feels that one cannot describe transition fairly in terms of a single point, but rather one should be able to define what percent of the time any point on the plate would be turbulent. He then develops a theory for predicting what percentage of the time each position on the plate will be turbulent, by considering not only the effect of spots formed at the point in question but also those that form upstream. Some rather complicated expressions result for the probability that a certain spot on the plate will be turbulent. Much additional experimental evidence is necessary to complete this theory, mainly data on the rate of spot production, the initial size of a turbulent spot, the rate at which a spot grows as it moves downstream, and how these parameters are affected by free stream turbulence.

Although this theory will undoubtedly aid in defining better the

conditions in the boundary layer through transition, the lack of definitive data does not make it too useful at the present time. This brief discussion was included here merely to indicate yet another method of defining transition and the conditions immediately surrounding it.

From this brief literature survey it can be concluded that with lower levels of turbulence at least the oscillations of the laminar layer play an integral part in the mechanism of transition. As pointed out by Schubauer and Skramstad, an understanding of transition is near mainly because the laminar oscillations are more fully understood. One may also conclude that transition does not occur instantaneously, but extends some distance along the plate. The purpose of this investigation is to supply information concerning the details of the flow in the transition region and how these conditions are altered by introducing turbulence of varying degree.

APPARATUS AND METHODS

THE TUNNEL

The experimental work was performed in the wind tunnel of the Research & Development Laboratories of the Kimberly-Clark Corporation. Sketches and a photograph of the tunnel are shown in Figures 2, 3, 4 & 5. The tunnel is of the single pass type and is placed in a long building composed of two quonset huts in tandem, each quonset being raised seven feet from the floor level to give a maximum ceiling height of seventeen feet. The tunnel has a 14 to 1 contraction ratio and test section dimensions of 20 x 30 inches and six feet long. The test section shape is rectangular with fillets in the corners as indicated in Figure 4. To eliminate a longitudinal static pressure gradient in the test section, the side walls are adjustable in fixed increments. Faired fillet pieces were placed at the end of the test section to eliminate a bad discontinuity at the diffuser entrance when the side walls were expanded. Figure 6 shows the pressure gradient plotted in the usual dimensionless form $\frac{\Delta P}{q}$ as a function of distance from the entrance of the test section for several settings of the side wall. The velocity for these tests was roughly 120 ft./sec. It is seen from Figure 6 that by moving each wall $1/2^{\circ}$ outward gives a pressure gradient uniform to within 1% of the dynamic head over the working length of the test section i.e. over the length of the plate. The velocity distribution, as measured in several transverse planes across the test section, was found to be uniform to within 0.5% over the working range of the jet.

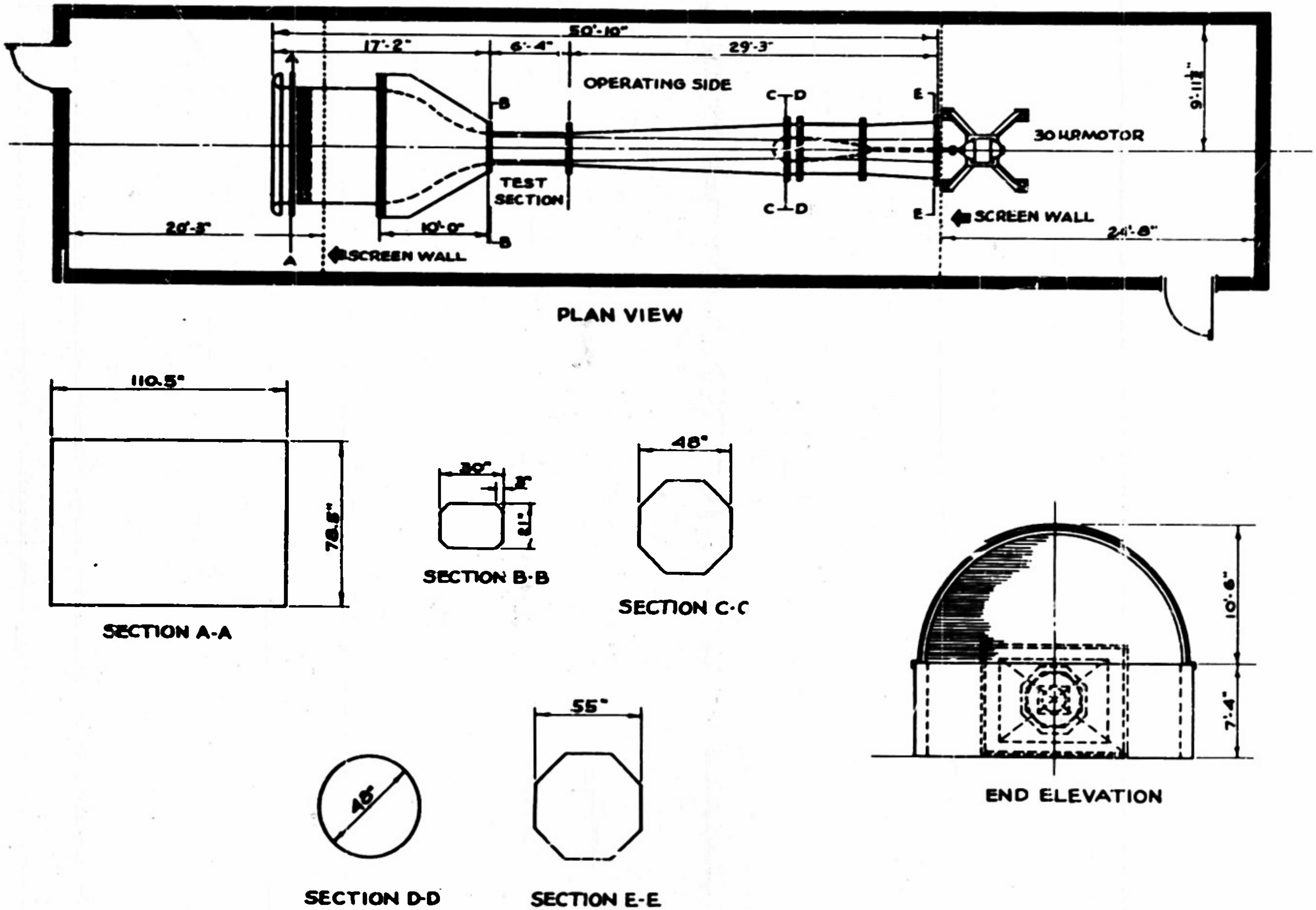


FIG. 2 ILLUSTRATION SHOWING TUNNEL INSTALLATION

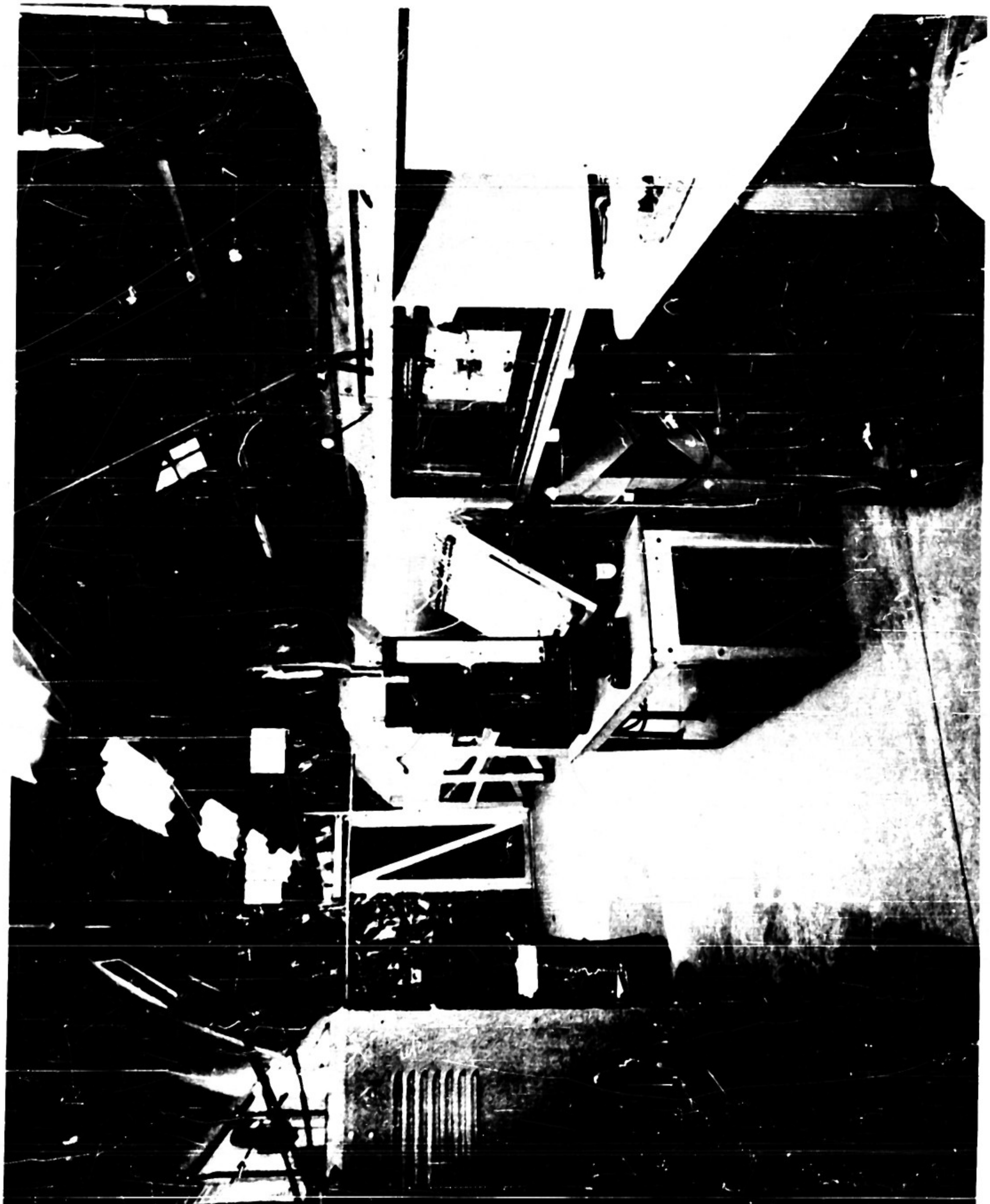


FIGURE 3—Photograph of tunnel.

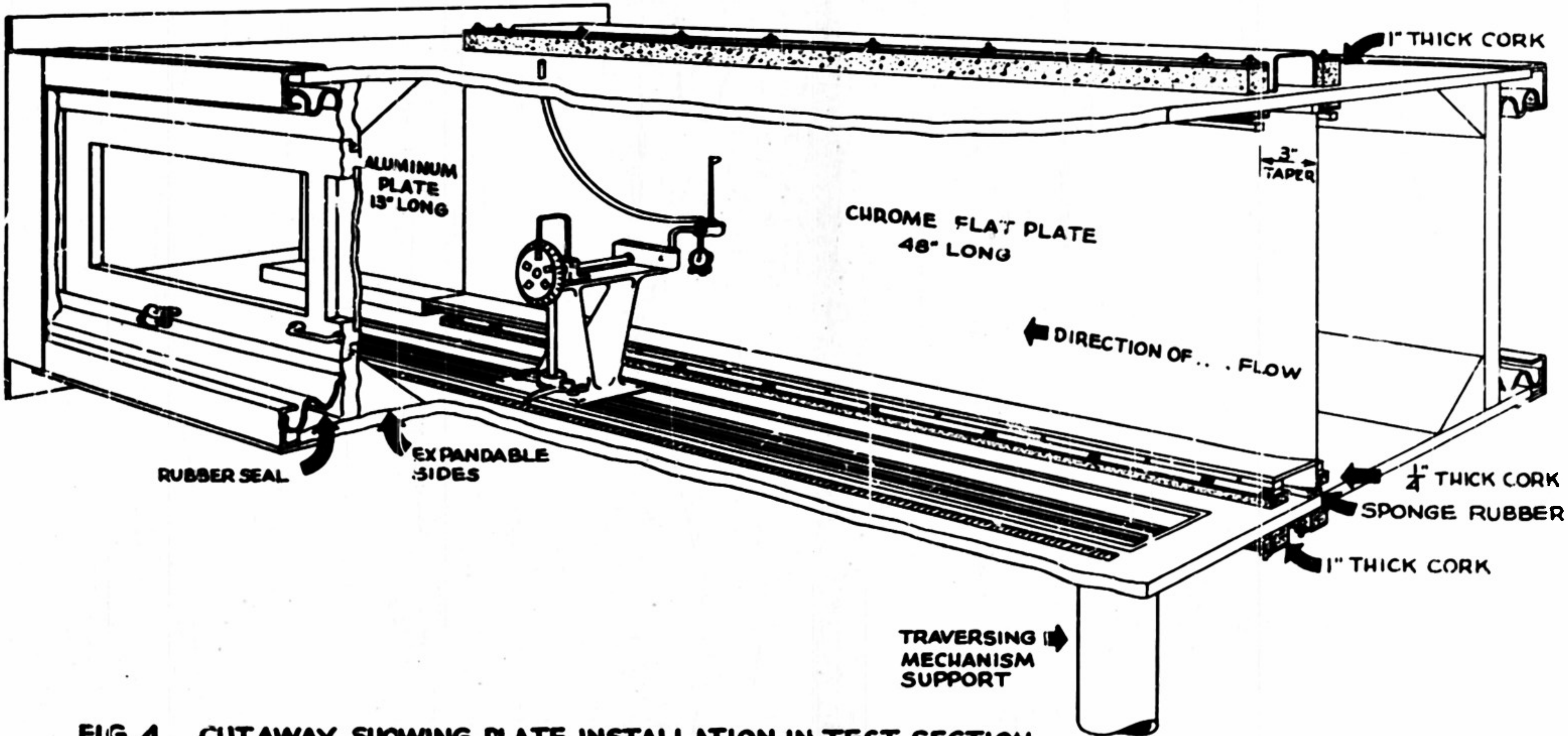


FIG. 4 CUTAWAY SHOWING PLATE INSTALLATION IN TEST SECTION

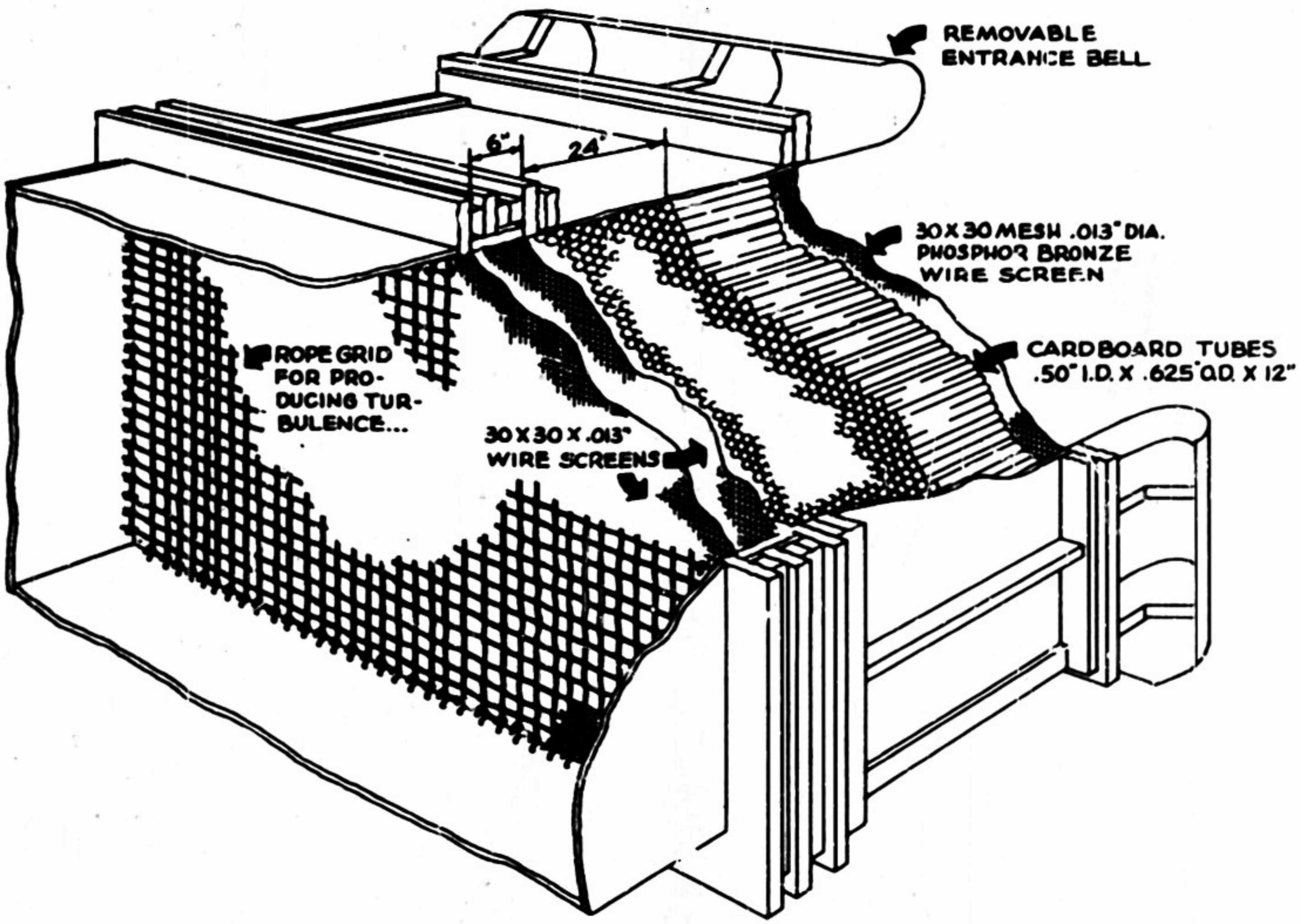


FIG. 5 CUTAWAY DRAWING SHOWING CONSTRUCTION OF TUNNEL INLET

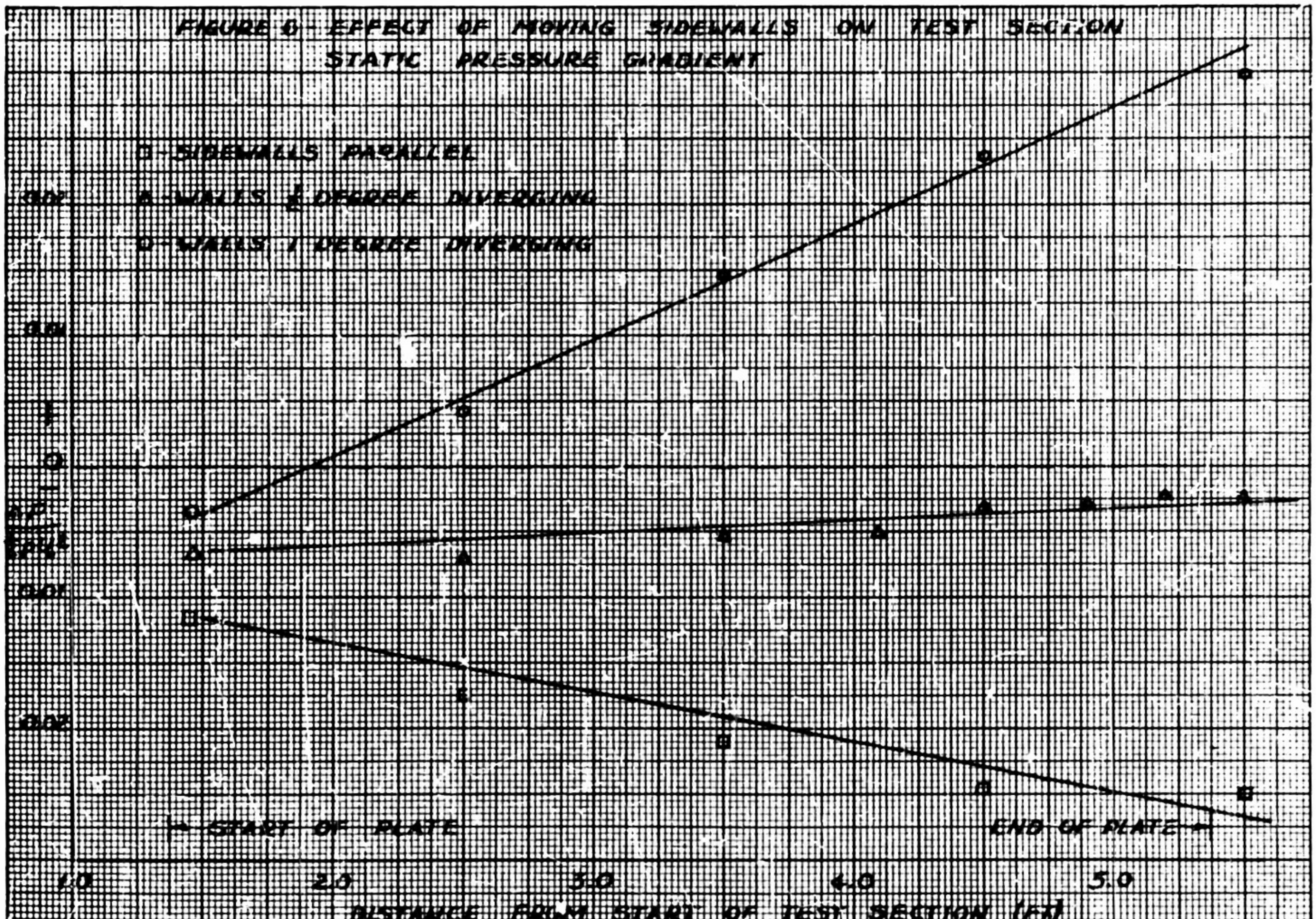


FIGURE 6 - EFFECT OF MOVING SIDEWALLS ON TEST SECTION STATIC PRESSURE GRADIENT

Some difficulty was experienced with the return flow of air because of the somewhat confined cross-sectional area in which the air returns to the tunnel entrance. Because of this confined area, it was felt paramount that the return flow be distributed over the entire cross section of the room and thus have as low a velocity as possible. To achieve this, two walls of 60 x 60 mesh wire were introduced across the quonset as indicated in Figure 2. Although these walls introduced a considerable pressure drop and limited the maximum test section velocity, they served to distribute very effectively the return flow and keep the return velocity low. To eliminate any remaining large scale irregularities in the incoming flow a 30 x 30 mesh screen with 0.013 inches diameter wire was placed immediately following the bellmouth. This screen was directly followed by a honeycomb of paper tubes with an I.D. of 0.5 inches, an O.D. of 0.625 inches and a length of 12 inches. Following the honeycomb was a short settling length as indicated in Figure 5 and two more 30 x 30 mesh screens. The flow in the tunnel was very uniform and free from pulsations in the main flow. The turbulence level was probably not as low as could have been achieved using more conventional mesh sizes, but procurability of the screens was the deciding factor in this case.

To change the turbulence level in the tunnel, rope grids with various diameter ropes were introduced immediately downstream of the last screen. Figure 7 shows a typical grid installation. Good control of turbulence was attained in this manner.

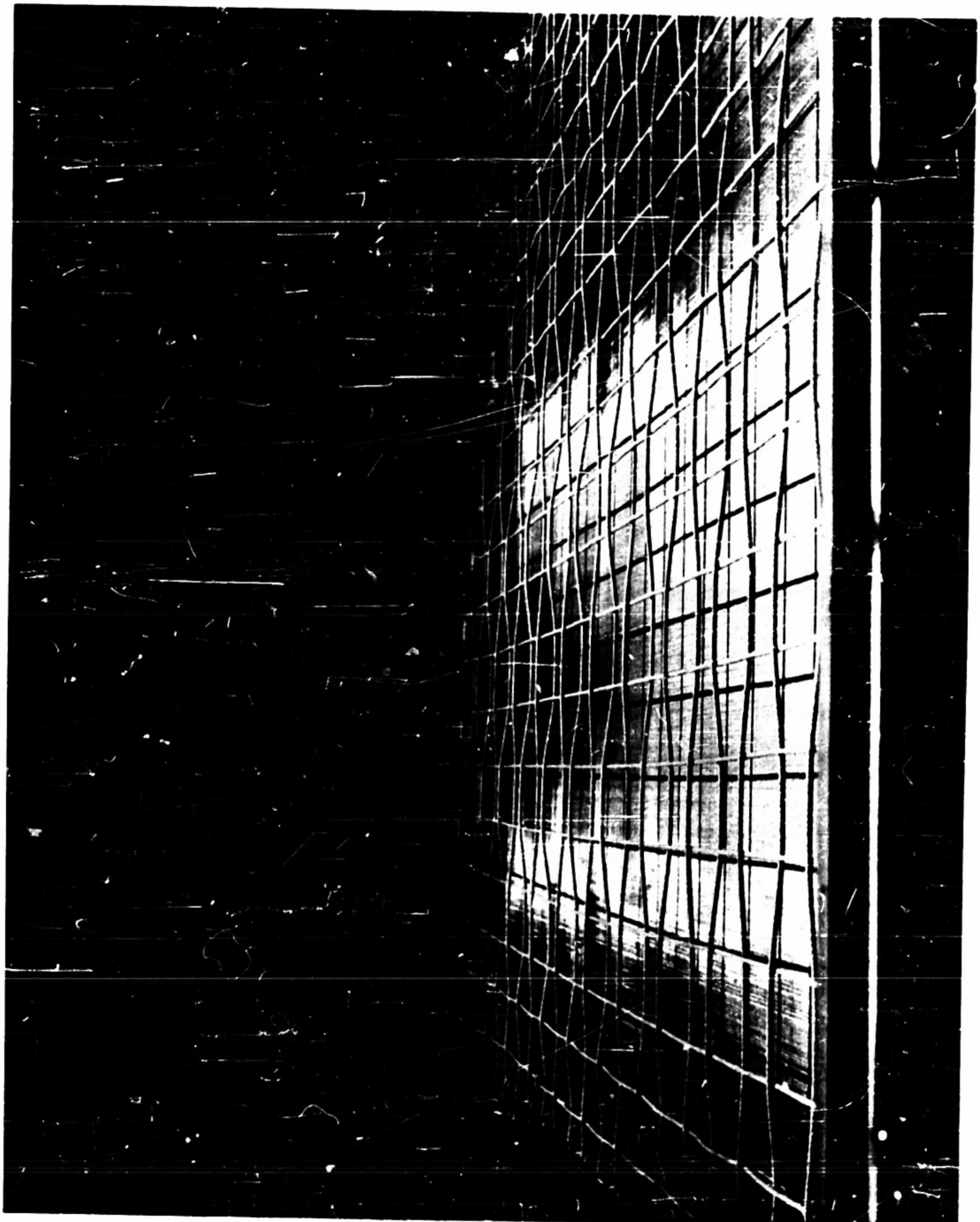


FIGURE 7—Photograph of rope grid installation.

HOT WIRE EQUIPMENT

The hot wire equipment was of the constant current type, the changes in voltage across the wire being an indication of the velocity changes. Included with the hot wire heating equipment and the amplifier, was the usual assortment of galvonometers, potentiometers, switching circuits, bridge circuits, etc. The amplifier is of the capacitance compensated type with a flat response from 0 to 10,000 cycles.

The wires used for u' measurements were of 0.00031 inch diameter tungsten wire supported as shown in Figure 8. The probes consisted of two brass strips separated by an insulating material. To each of the strips was fastened a thin support wire. The tungsten wire was welded to the support wires by the technique described in Ref. 9.

In making v' measurements, an x-wire probe, as shown in the Figure 8, was used. The wire used was 0.0002 inches platinum. The platinum wire was soft-soldered between common sewing needles which are clearly evident in the photograph. The entire assembly behind the needles was encased in a hollow 3/8" O.D. tube, some six inches long, as shown in Figure 8.

To traverse the wire, the carriage assembly shown in Figure 9 was used. A portion of the tunnel floor was removed and the metal bed, on which the carriage traveled, inserted. A long lead screw, operated externally, gave the entire carriage longitudinal motion and the short precision lead screw shown in the picture, also operated from an external handwheel through a shaft and gear system, gave the probe mount crosswise motion. Although the probe wire is rather close to

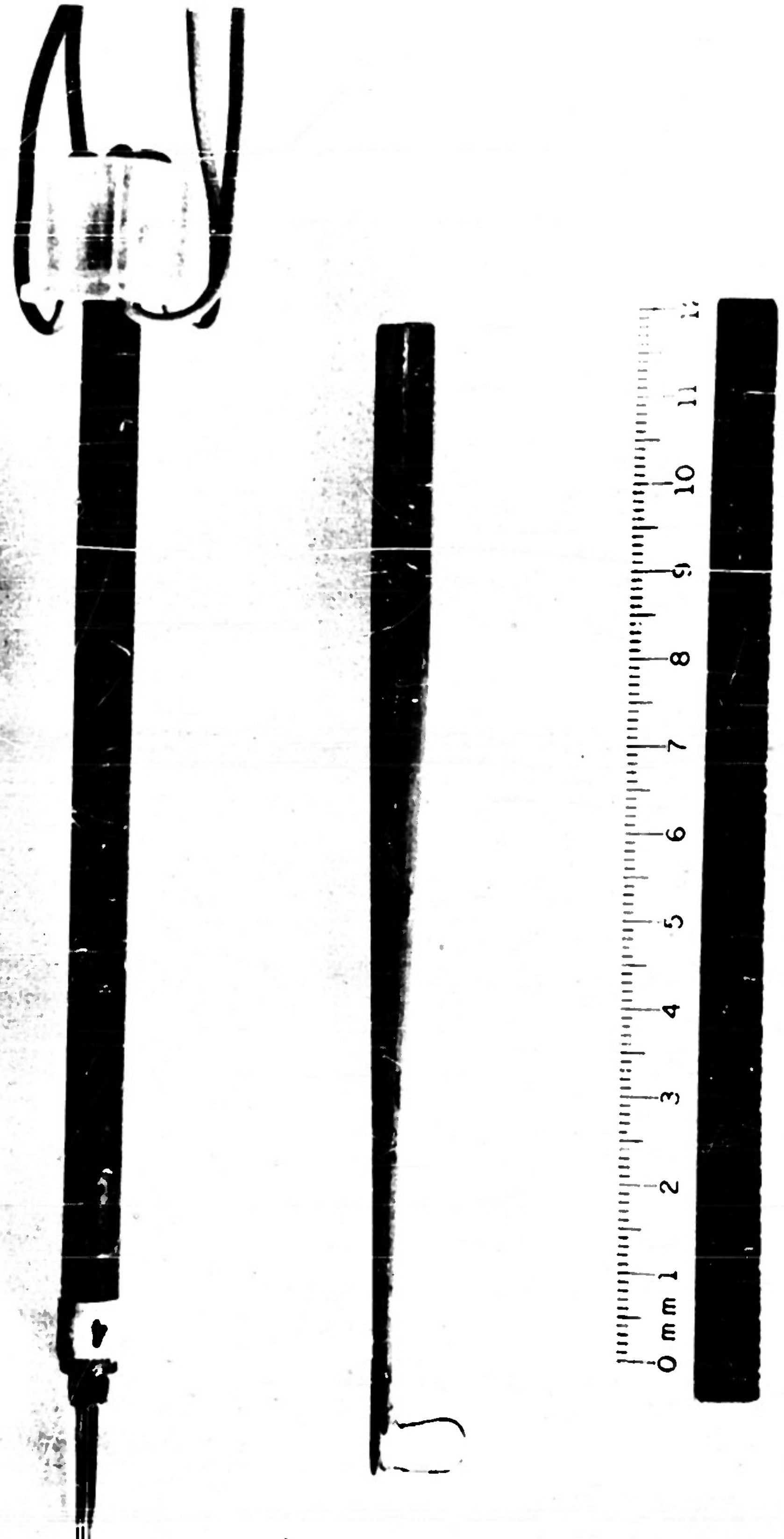


FIGURE 8—Photograph of hot wire probes used in tests.

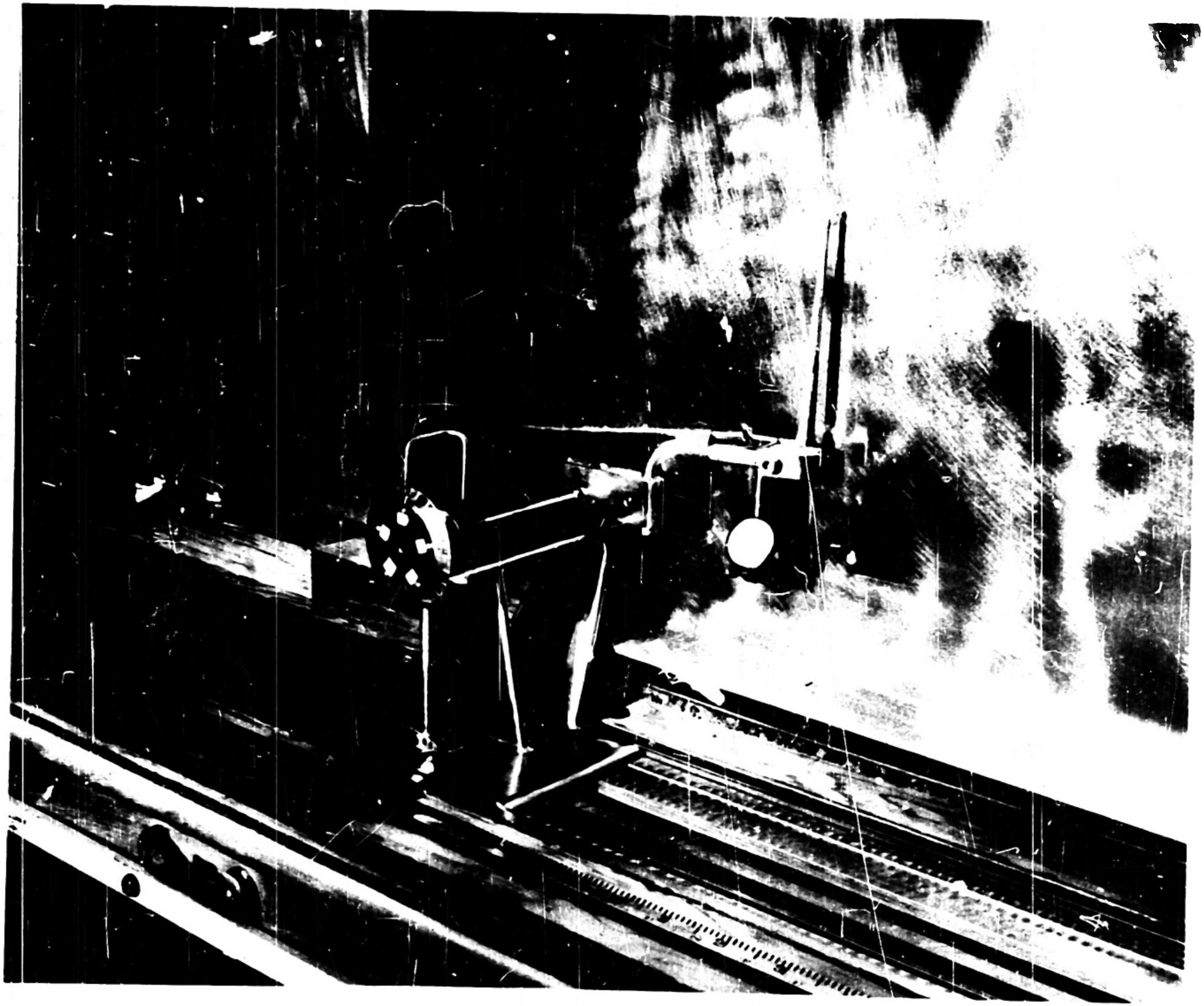


FIGURE 9—Photograph of plate and carriage.

the carriage as shown in the photograph, no observable pressure field effects were encountered.

To position the probe wire relative to the plate, feeler gauges were used and the reading on the dial indicator attached to the probe holder (as seen in the photo of Figure 9) noted. The dial indicator was set at the same reading and it was assumed that the probe wire was always the same distance from the plate as the probe was moved along the plate. This, of course, assumes that there is no transverse curvature in the plate. Actually it was found that the distance from the probe to the plate varied ± 0.001 inch as the probe was moved down the plate. This was not thought serious.

The compensation constant of the wire in use was set by superimposing an alternating current of constant current and different frequencies on the heating current, plotting the output of the wire as a function of frequency, and from this curve, calculating the best time constant. This method, although somewhat time consuming, was found to be very satisfactory.

To measure the distribution of fluctuation energy with frequency, a panoramic sonic analyzer was used. This instrument, which is similar to an oscilliscope, samples once a second the frequency spectrum from 20 to 20,000 cycles and automatically plots the output as a function of frequency. To get an average picture of the spectrum, a long-time photograph of the face of the instrument was taken and the image "burned in" the photographic paper. Interpretation of these photo-

graphs was somewhat difficult, but by connecting points of equal optical intensity it is felt that a fair picture of the frequency spectrum results. It is certainly a simple method and much less time consuming than the usual band-pass filter method and shows adequately the rather large changes in spectrum which occur in boundary layer transition.

VELOCITY MEASUREMENTS

To measure the velocity at the surface of the plate the dynamic pressure was measured using the impact tube shown in Figure 10. This tube consists of a hypodermic needle of roughly 0.0010 I.D. and 0.0020 O.D. The needle was fastened into the end and perpendicular to the axis of a 3/16" hollow brass tube. A simple mounting arrangement permitted the probe to be mounted in the traversing carriage. The static pressure was measured using the static probe also shown in Figure 10. This probe consisted of a long cylinder with an ogive-shaped nose and pressure ports around the cylinder 8 diameters back from the tip of the nose. This probe was used for measuring the test section static pressure gradient and also as a reference for the impact tube when measurements were being made with the latter near the surface of the plate. For these latter tests, the static probe was held in a fixed position near the center of the area on the working side of the plate and about halfway down the test section.

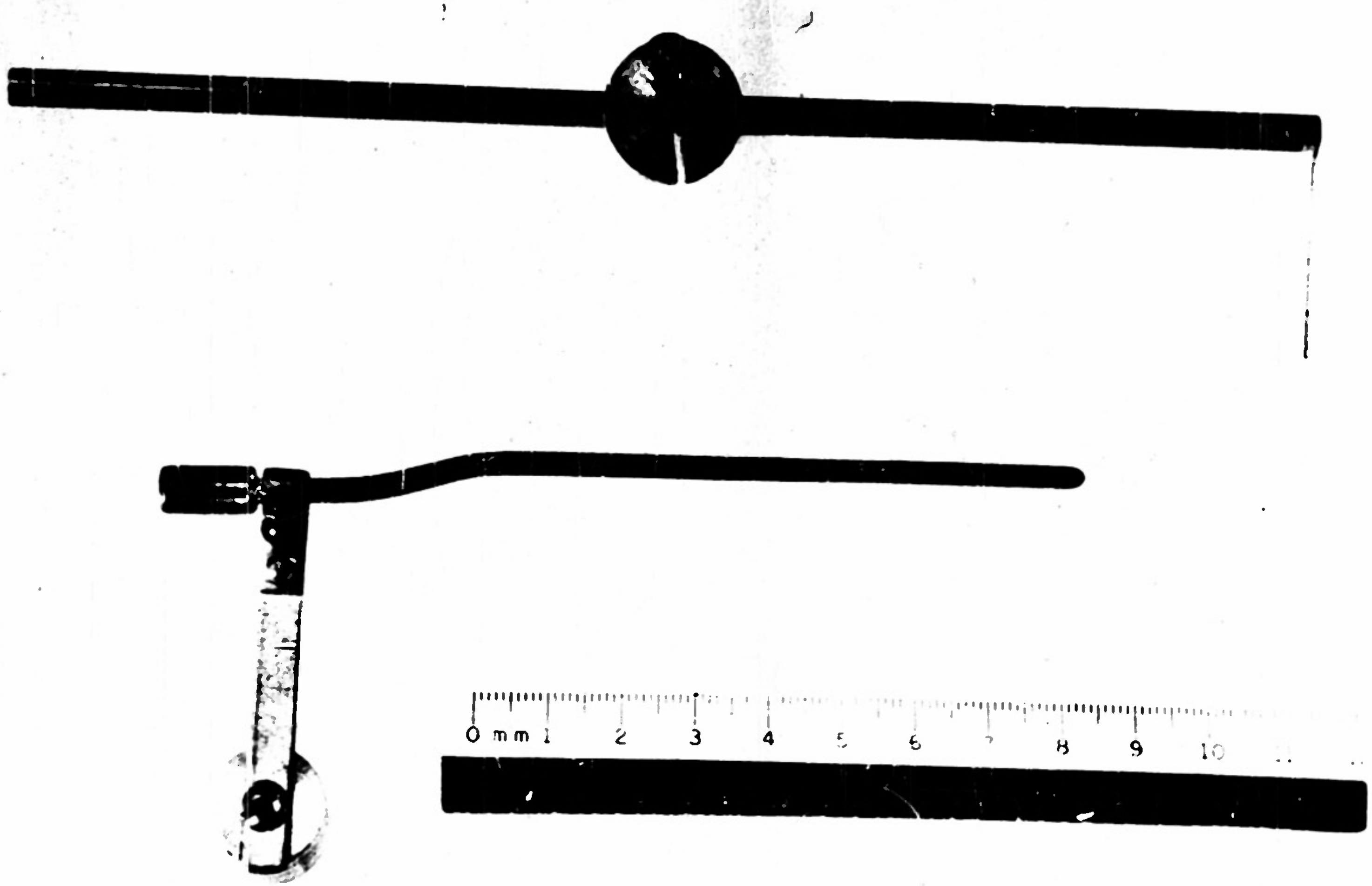


FIGURE 10—Photograph of pitot tubes.

FLAT PLATE

The flat plate was 1/4" sheet steel, prepared on a surface grinder, hand polished and then chrome plated. A surface finish of 10 to 15 r.m.s. micro-inches was achieved.

The leading edge of the plate was well sharpened and a smooth transition formed some 3" from the edge. No particular contour was followed, but no discontinuity in the surface at the junction between the curve and the flat portion was noticeable to the touch.

The plate was supported both top and bottom by a row of bolts passing through the plate. To isolate the plate from tunnel vibration, strips of cork were placed under the supporting strips, and strips of rubber between the plate and the ceiling, and the plate and floor of the tunnel, as indicated in Figure 4. The cork strips seemed to effectively damp most of the tunnel vibration.

The plate did not at first extend back to the end of the test section, but it was soon observed that the balance between the flow on either side of the plate was upset when the carriage of the traversing mechanism was moved back behind the trailing edge of the plate. To correct this, an extension was added to the plate of 1/4" aluminum. No measurements were made, of course, on the aluminum extension but no further difficulty with the balance of air on the two sides of the plate was encountered.

RESULTS AND DISCUSSION

Figure 11 shows a typical series of velocity measurements a fixed distance away from the plate (approximately 1/2 diameter of hypodermic needle shown in Figure 8) as a function of distance down the plate. The velocity was referred to a fixed static probe elsewhere in the test section. The sudden rise in pressure, caused by the "washing away" of the laminar profile by the steeper turbulent profile is usually defined as the transition point, although as will be shown below, transition very definitely has its beginnings upstream some distance from this point. Immediately following the peak of this curve is the decrease in velocity, a fixed distance from the surface, caused by the natural growth of the turbulent layer. The peak of this curve will be called the end of transition.

Figure 12 shows the beginning and end of the transition region for several levels of free stream turbulence. Because this portion of the work was not felt to be the most important, only u' and v' in the free stream were measured and it was assumed that w' equals v', as is usually roughly the case. Also shown are data taken by Schubauer and Skramstad (Ref. 6). Agreement is seen to be fair except the point for the lowest tunnel turbulence level 0.013. No explanation for this discrepancy has been found except that possibly the distribution with frequency of the disturbances present was not the same in the two cases. As shown by Schubauer and Skramstad, a good portion of the fluctuation energy picked up by their hot wire at low tunnel turbu-

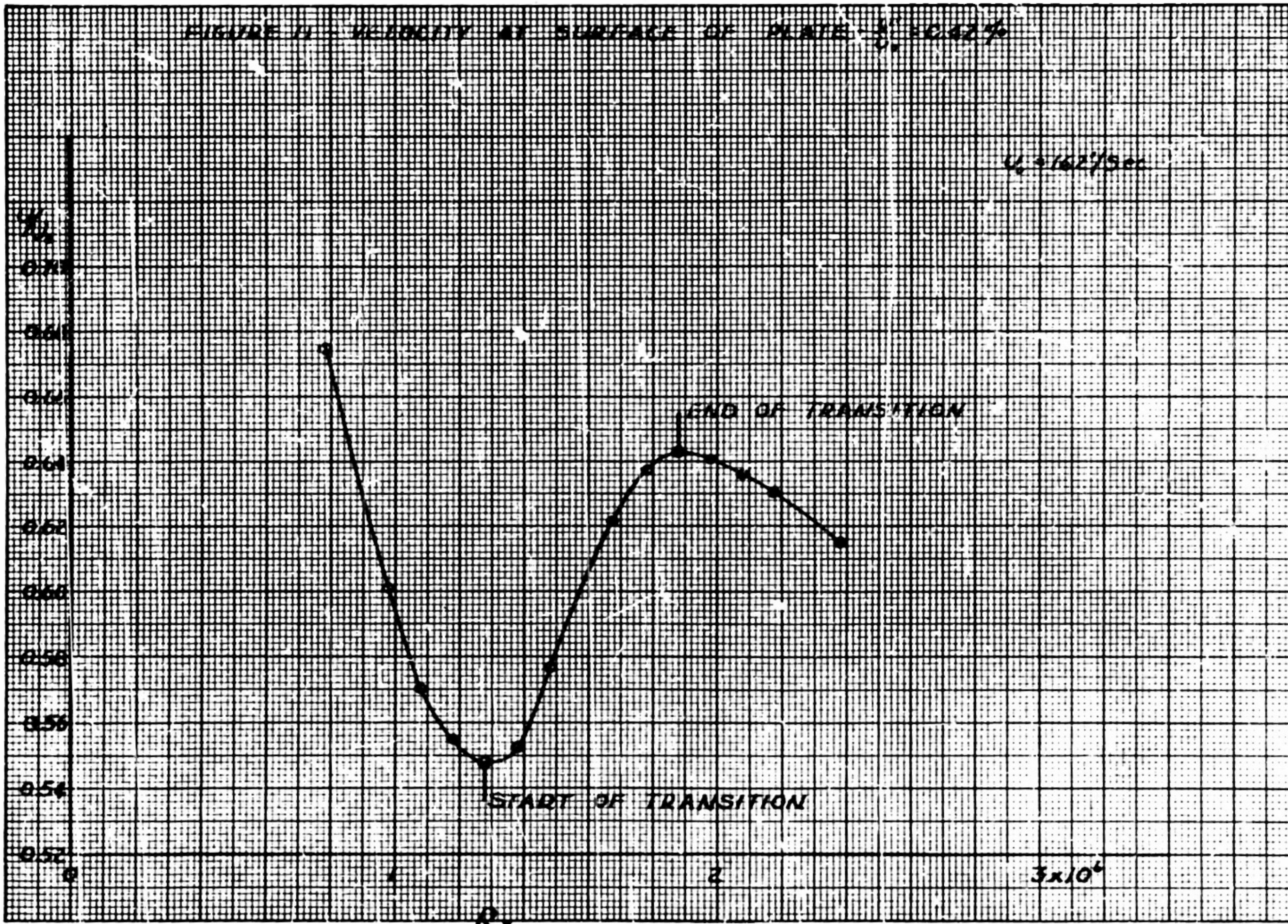


FIGURE 11 - VELOCITY AT SURFACE OF PLATE $\frac{u'}{U_0} = 0.42\%$

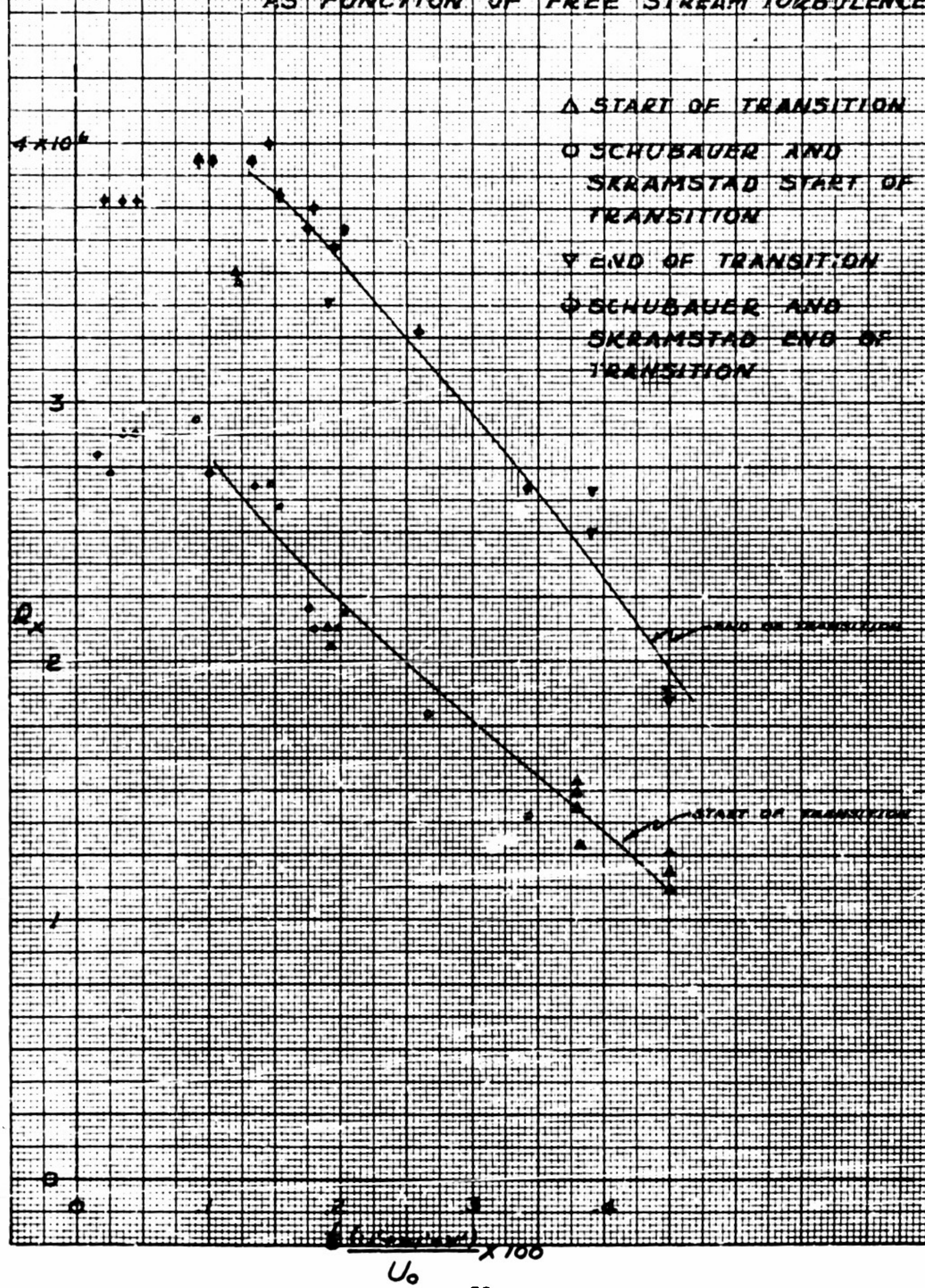


FIGURE 12 - BEGINNING AND END OF TRANSITION AS FUNCTION OF FREE STREAM TURBULENCE

lence was caused by propeller noise. As pointed out in their report and as will be discussed below, with low turbulence, the laminar boundary layer is very susceptible to disturbances of certain narrow frequency bands and more likely than not, eventual transition at these low turbulence levels is caused not by the free stream turbulence but rather by the noise and vibration present in the tunnel circuit. Quite possibly the distribution of energy with frequency in our case was markedly different than in their case but no work was accomplished along these lines as it was not felt that the divergence of our data from theirs warranted it.

Of more interest than the point of transition or the length of the zone are the fluctuations in the laminar layer, their eventual decay into turbulence, and the relation of these phenomona to the modification of the laminar velocity distribution.

Figure 13 shows photographs of a single sweep on an oscilliscope taken by triggering the sweep for a single pulse in front of the open camera lens. The photographs were taken with the probe 0.010 inches away from the surface. The relative gain for each picture is as indicated below the picture. It is seen that the oscillations discussed above are truly present and are seen to build up in amplitude and are then broken down into a more typically turbulent configuration as seen in the last photograph. Comparing these photographs to the curves of Figure 12, it is observed that transition truly has its beginnings some distance before the turbulent layer replaces the laminar layer.

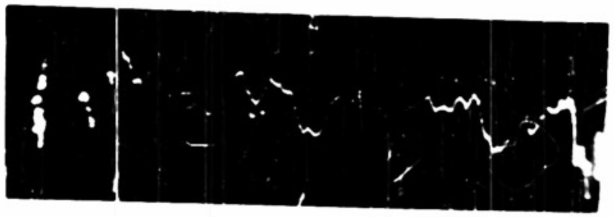
FIGURE 13

OSCILLISCOPE TRACES OF HOT WIRE OUTPUT IN BOUNDARY LAYER

$U_o = 162'/\text{sec.} \quad \frac{u'}{U_o} = 0.25\%$

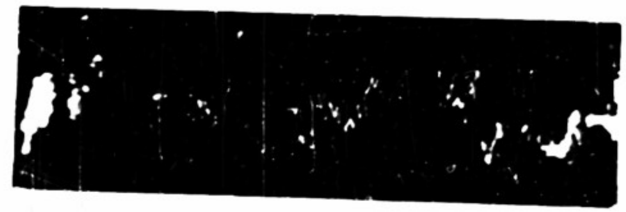
PROBE 0.010 INCHES FROM SURFACE

TIME BETWEEN BREAKS 1/60 SEC.



$R_x = 0.46 \times 10^6$
RELATIVE MAGNIFICATION = 4

$R_x = 0.76 \times 10^6$
RELATIVE MAGNIFICATION = 3

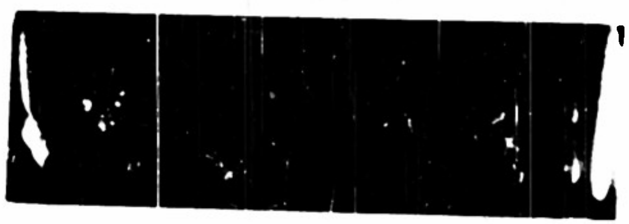


$R_x = 0.86 \times 10^6$
RELATIVE MAGNIFICATION = 2

$R_x = 1.06 \times 10^6$
RELATIVE MAGNIFICATION = 2

$R_x = 1.16 \times 10^6$
RELATIVE MAGNIFICATION = 1.5

$R_x = 1.36 \times 10^6$
RELATIVE MAGNIFICATION = 1



$R_x = 1.56 \times 10^6$
RELATIVE MAGNIFICATION = 1.0

$R_x = 1.86 \times 10^6$
RELATIVE MAGNIFICATION = 1

$R_x = 2.36 \times 10^6$
RELATIVE MAGNIFICATION = 2

Concerning the fluctuation energy increase in the laminar layer, Figure 14 shows a plot of $\frac{u'}{U}$ for the same conditions as the velocity plot of Figure 11. Included for orientation purposes is the transition point. Once again the probe was a fixed distance (0.010 in.) away from the plate. It must be remembered that with the probe a fixed distance from the plate the relative position of the wire in the boundary layer is changing, as the probe is moved down the plate. As shown by Schubauer and Skramstad (Ref. 6) the amplitude of the Tollmien-Schlichting waves is a function of $\frac{y}{\delta}$. Nevertheless, for comparing conditions at different levels of free stream turbulence, this type of presentation is instructive.

From the plot of Figure 14, it is noted that the increase in fluctuation energy is not a sudden phenomena, concurrent with the modification of the velocity profile, but is rather a more gradual process. To fully understand what occurs as the fluctuation energy increases, it is necessary to examine the frequency spectrum of the fluctuations.

MEASUREMENT OF SPECTRUM

Figure 15 shows a series of photographs of the panoramic analyzer which, in effect, are plots of output versus frequency of the compensated amplifier looking at the hot wire which once again is 0.010 inches from the plate and a distance from the leading edge as indicated by the Reynolds number under the photo. Initially the hot wire notes low amplitude waves, some of which appear to be stray electrical or mechanical pick-up. As the R_x value is increased, it is seen

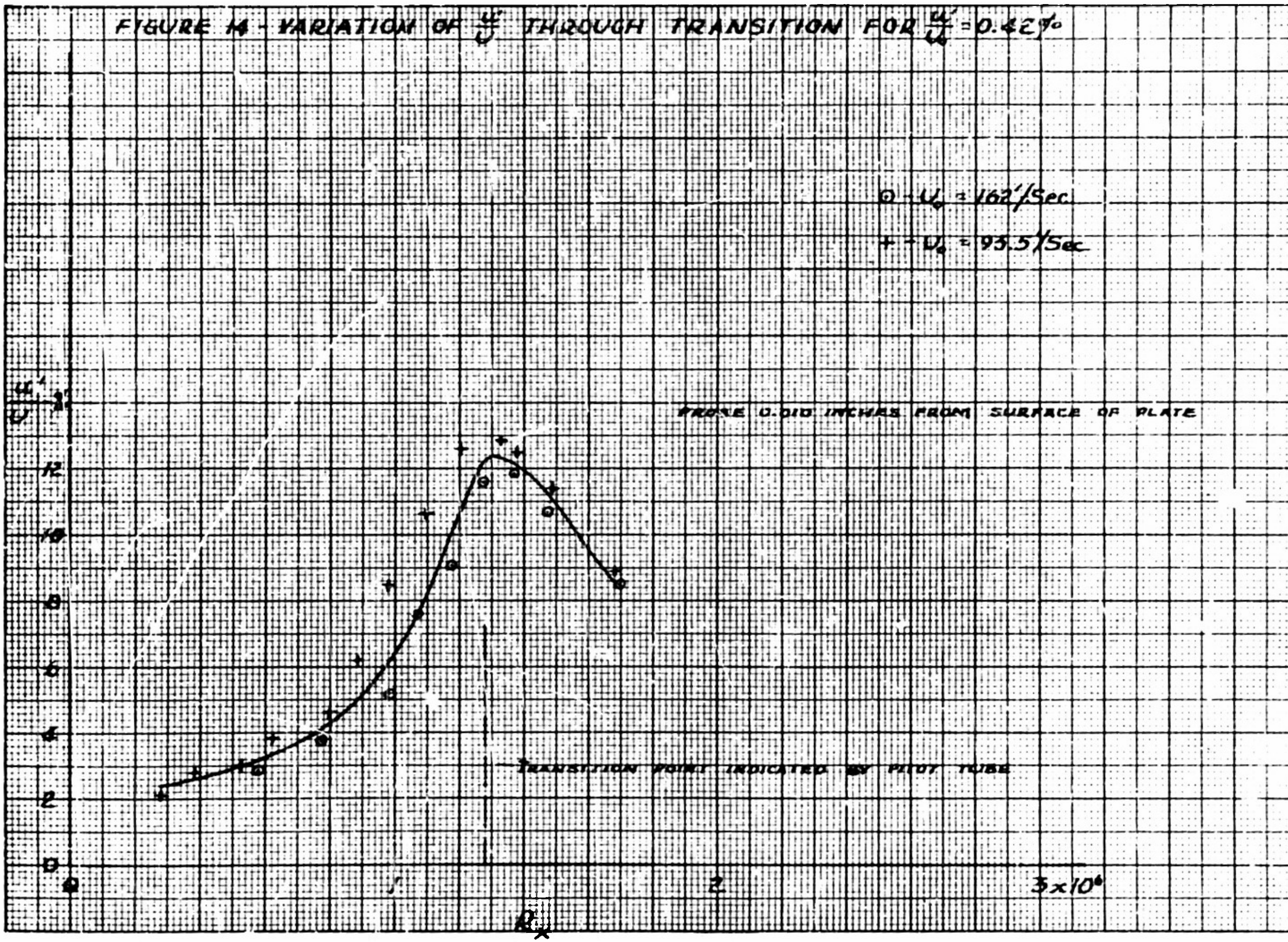
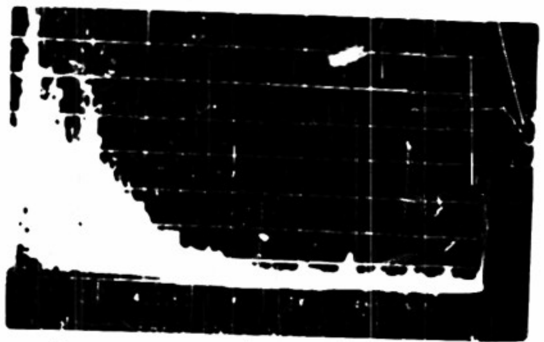


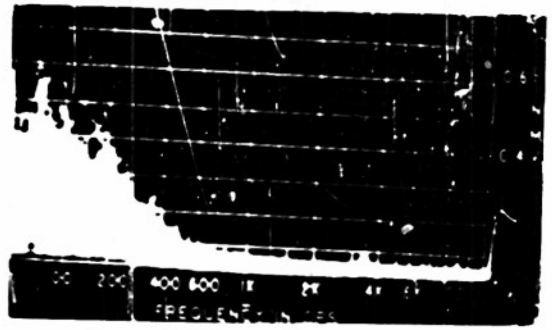
FIGURE 14 - VARIATION OF $\frac{u'}{U}$ THROUGH TRANSITION FOR $\frac{u'}{U_0}$ = 0.42%
⊙ - U_0 = 162'/Sec.
+ - U_0 = 93.5'/Sec.
PROBE 0.010 INCHES FROM SURFACE OF PLATE
TRANSITION POINT INDICATED BY PITOT TUBE
$\frac{u'}{U}$ %
12
10
8
6
4
2
0
0
1
2
3x10⁶
R_x

FIGURE 15

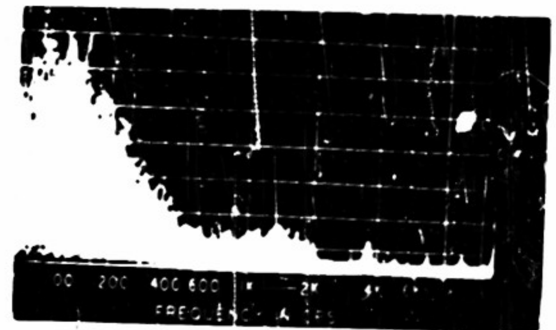
PHOTOGRAPHS OF SPECTRUM DEVELOPMENT. OUTPUT OF HOT WIRE AS FUNCTION OF FREQUENCY $\frac{u'}{U_o} = 0.42\%$ U_o = 162'/sec.



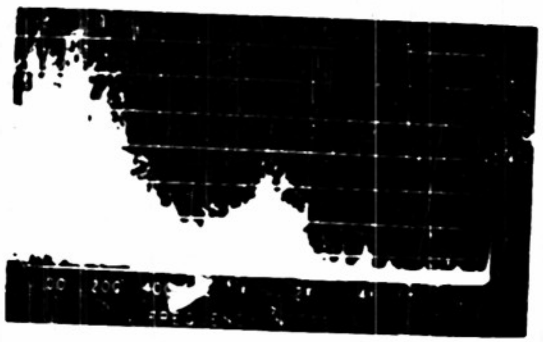
$R_x = 0.37 \times 10^6$
RELATIVE MAGNIFICATION = 5.3



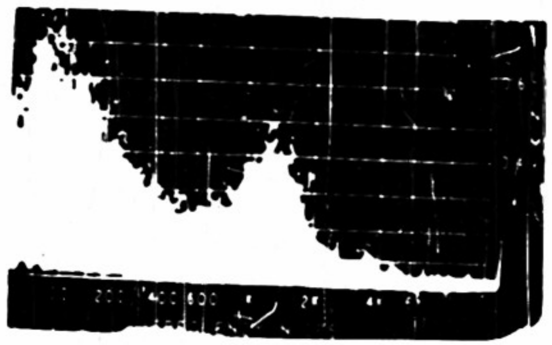
$R_x = 0.57 \times 10^6$
RELATIVE MAGNIFICATION = 2.65



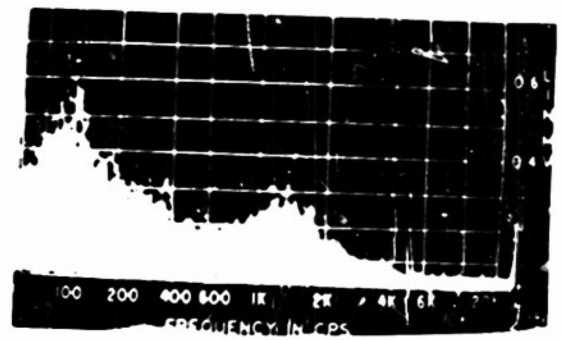
$R_x = 0.77 \times 10^6$
RELATIVE MAGNIFICATION = 2.65



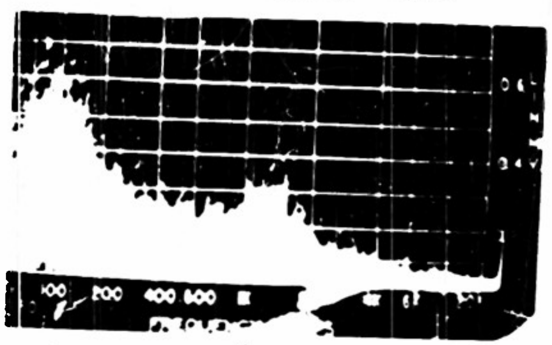
$R_x = 0.87 \times 10^6$
RELATIVE MAGNIFICATION = 2.65



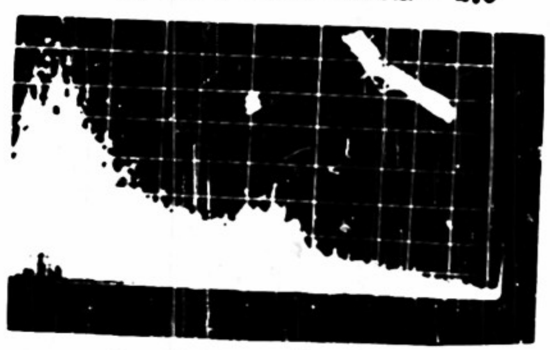
$R_x = 0.97 \times 10^6$
RELATIVE MAGNIFICATION = 2.0



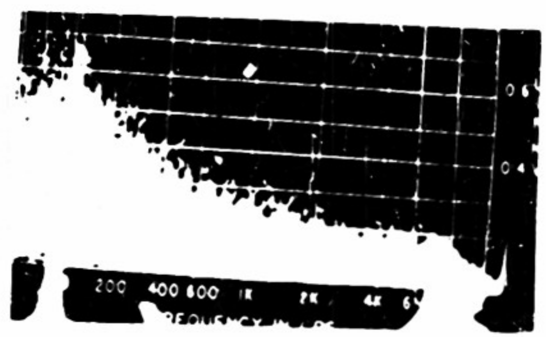
$R_x = 1.07 \times 10^6$
RELATIVE MAGNIFICATION = 1



$R_x = 1.17 \times 10^6$
RELATIVE MAGNIFICATION = 1



$R_x = 1.27 \times 10^6$
RELATIVE MAGNIFICATION = 1



$R_x = 1.67 \times 10^6$
RELATIVE MAGNIFICATION = 2

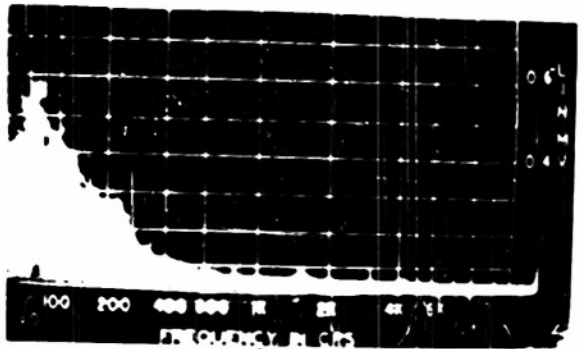
that a particular frequency is picked up by the boundary layer and amplified. The amplification continues for some distance down the plate, but the amplified frequency eventually becomes more and more distorted, as the band of frequencies which are amplified becomes wider and wider and it appears that the rest of the spectrum "feeds" on the amplified frequency, in much the same way that a turbulent wake is developed from a vortex street as discussed by Roshko (Ref. 10). The pattern of development was observed in all the tests made with the range of free stream turbulence used (0.13% to 0.5%); however, important differences showed up as the free stream turbulence and the free stream velocity were changed. These differences, discussed below, give a good deal of insight into the mechanism of transition.

EFFECT OF VELOCITY

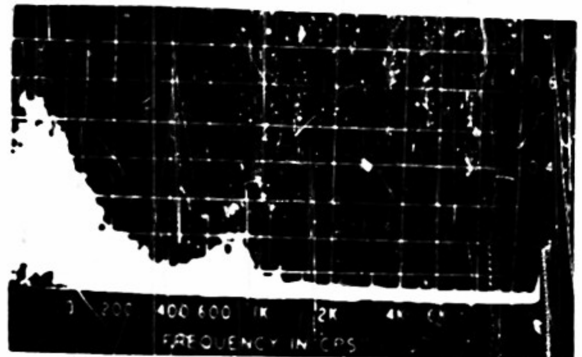
Figure 16 shows a series of photographs taken at a lower velocity than those presented above, but with the same grid configuration and thus nearly the same value of free stream turbulence. The difference which immediately is obvious is that the frequency which is amplified is lower than with the higher velocity. Referring to Figure 1, however, it is seen that $\frac{\beta_r \nu}{U_0^2}$ is the important parameter in determining which frequency will be amplified. Calculation of this frequency parameter showed it to be nearly constant for the same conditions of turbulence but different velocities.

FIGURE 16

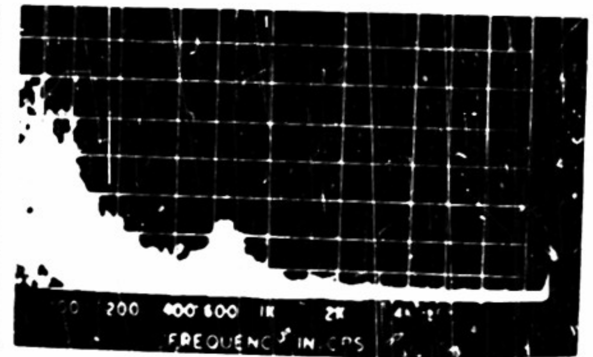
PHOTOGRAPHS OF SPECTRUM DEVELOPMENT. OUTPUT OF HOT WIRE AS FUNCTION OF FREQUENCY $\frac{u'}{U_0} = 0.42\%$ $U_0 = 93.5'/sec.$



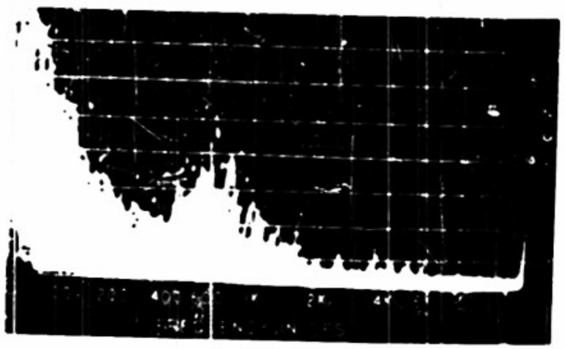
$R_x = .272 \times 10^6$
RELATIVE MAGNIFICATION = 3.35



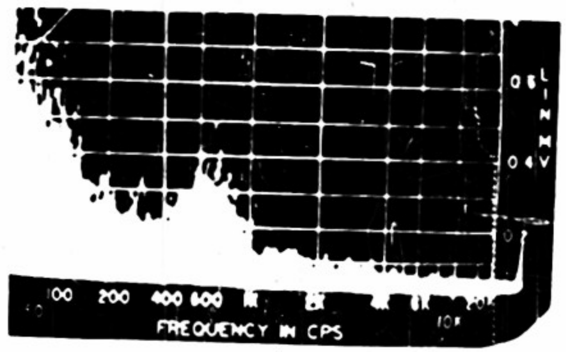
$R_x = 0.734 \times 10^5$
RELATIVE MAGNIFICATION = 1.4



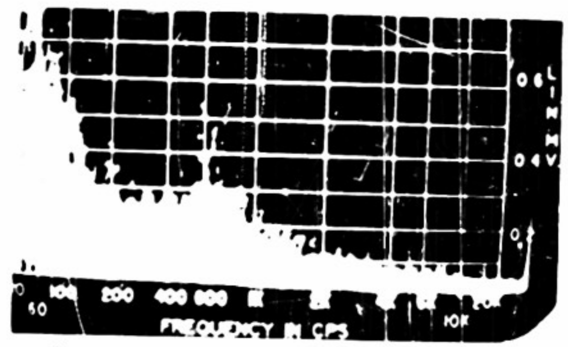
$R_x = 0.791 \times 10^6$
RELATIVE MAGNIFICATION = 1.4



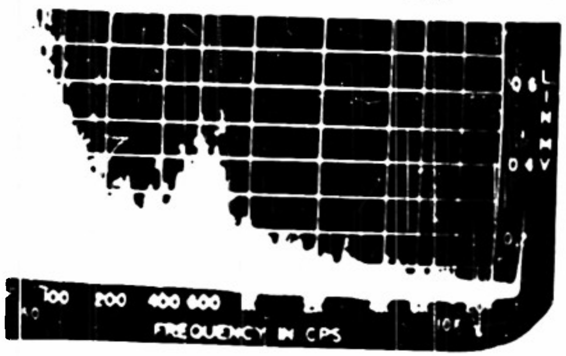
$R_x = 0.906 \times 10^6$
RELATIVE MAGNIFICATION = 1.4



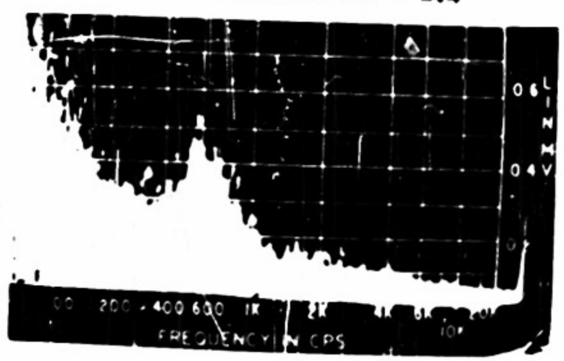
$R_x = 0.965 \times 10^6$
RELATIVE MAGNIFICATION = 1.4



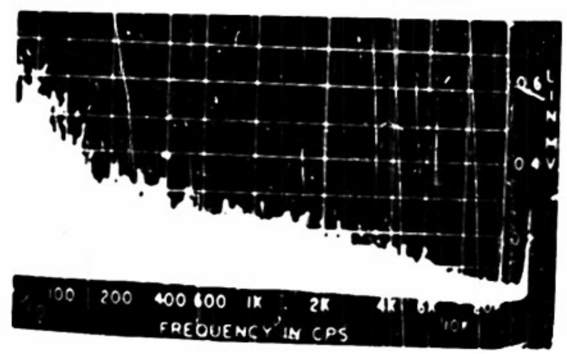
$R_x = 1.02 \times 10^5$
RELATIVE MAGNIFICATION = 1.4



$R_x = 1.076 \times 10^6$
RELATIVE MAGNIFICATION = 1.4



$R_x = 1.134 \times 10^6$
RELATIVE MAGNIFICATION = 1.



$R_x = 1.88 \times 10^6$
RELATIVE MAGNIFICATION = 1.4

EFFECT OF FREE STREAM TURBULENCE ON OSCILLATIONS AND EVENTUAL TRANSITION

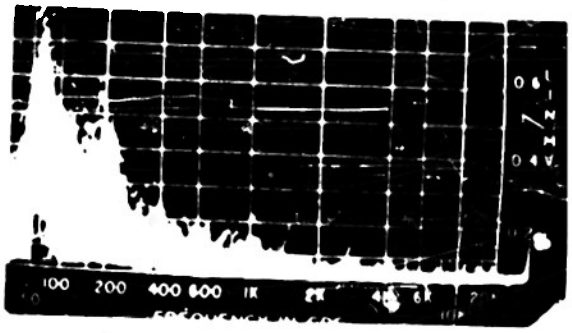
Figure 17 shows the development of oscillations for the lowest tunnel turbulence used 0.13% and Figure 16 the same information for the highest turbulence $\frac{u'}{U_o} = 0.42\%$. The speed was in both cases the same. Some important differences immediately are obvious: With higher turbulence, the frequency which the boundary layer picks up is quite a bit higher than with the lower turbulence and even more important the oscillation is extremely pure. With the higher turbulence there is almost immediately, on the appearance of the oscillations, a feeding of energy from the central frequency to the surrounding frequencies. This process, with the higher level of free stream turbulence, leads to transition almost immediately after the appearance of the waves. With lower turbulence, however, the oscillations are very pure, and there is very little transfer of energy to surrounding frequencies.

Concerning the frequency which the laminar layer amplifies, it was pointed out in the introduction that the frequency present at the point of observation is a function of conditions at that point, i.e., the frequency present will be that frequency which at that particular boundary layer Reynolds number value has received the most amplification up to that point. This implies that the frequency should be constantly changing as the hot wire probe is moved down the plate. Schubauer and Skramstad, using Schlichting's method of calculating the theoretical amplification, concluded that the frequency data, when plotted on the curve of neutral stability, should be inside the loop,

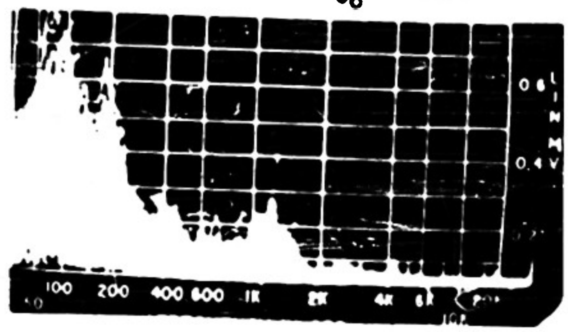
FIGURE 17

$U_o = 162'/sec.$

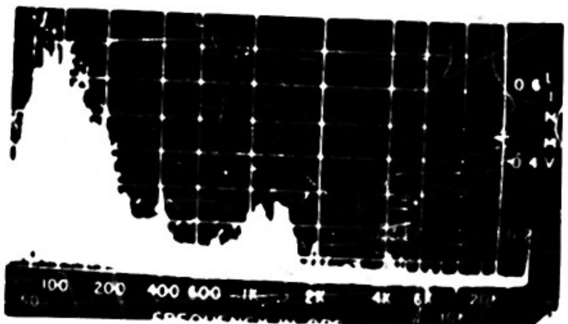
DEVELOPMENT OF SPECTRUM. HOT WIRE OUTPUT AS FUNCTION OF FREQUENCY. $\frac{u'}{U_o} = 0.13\%$



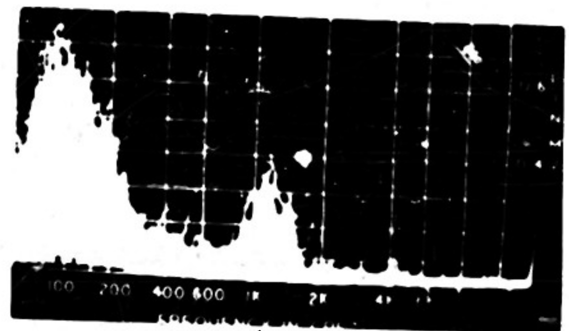
$R_x = 1.17 \times 10^6$
RELATIVE MAGNIFICATION = 5



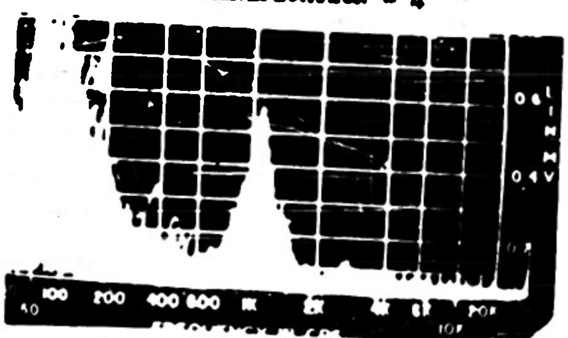
$R_x = 1.37 \times 10^6$
RELATIVE MAGNIFICATION = 5



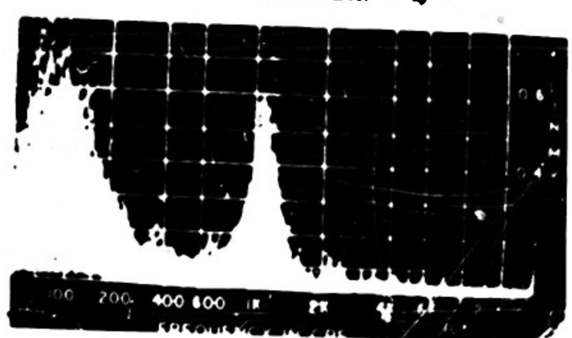
$R_x = 1.47 \times 10^6$
RELATIVE MAGNIFICATION = 4



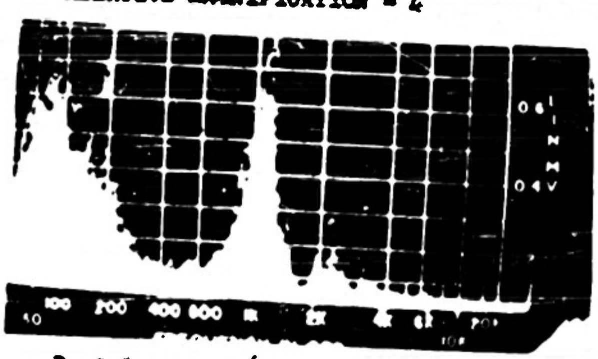
$R_x = 1.57 \times 10^6$
RELATIVE MAGNIFICATION = 4



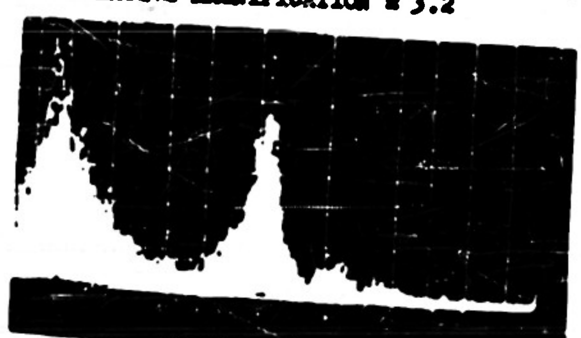
$R_x = 1.67 \times 10^6$
RELATIVE MAGNIFICATION = 4



$R_x = 1.77 \times 10^6$
RELATIVE MAGNIFICATION = 3.2

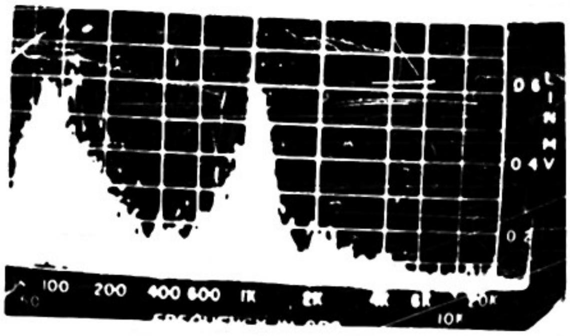


$R_x = 1.87 \times 10^6$
RELATIVE MAGNIFICATION = 2.3

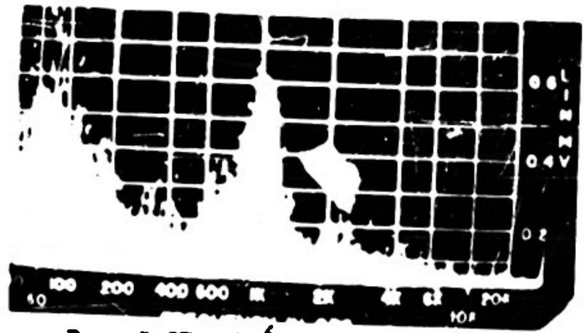


$R_x = 1.97 \times 10^6$
RELATIVE MAGNIFICATION = 2.0

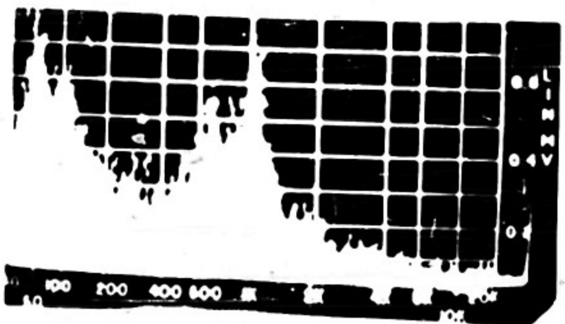
FIGURE 17 (CONT.)



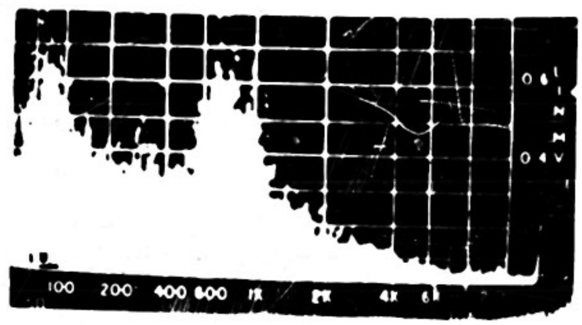
$R_x = 2.17 \times 10^6$
RELATIVE MAGNIFICATION = 2.0



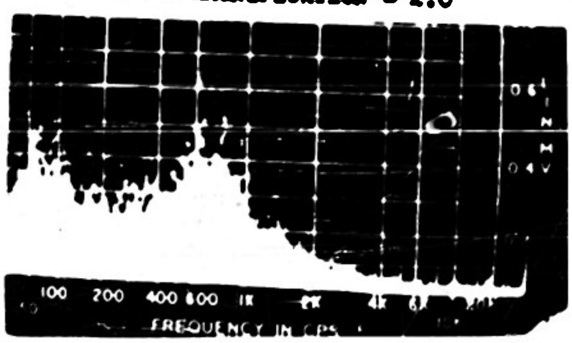
$R_x = 2.27 \times 10^6$
RELATIVE MAGNIFICATION = 2.0



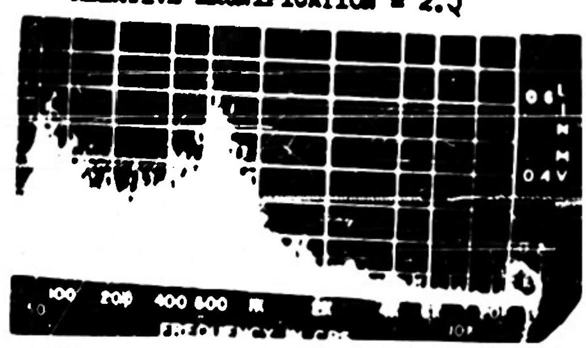
$R_x = 2.47 \times 10^6$
RELATIVE MAGNIFICATION = 2.0



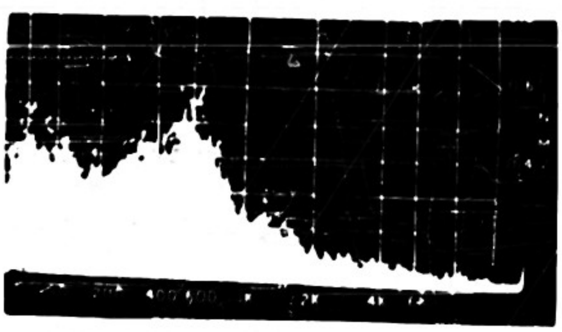
$R_x = 2.77 \times 10^6$
RELATIVE MAGNIFICATION = 2.0



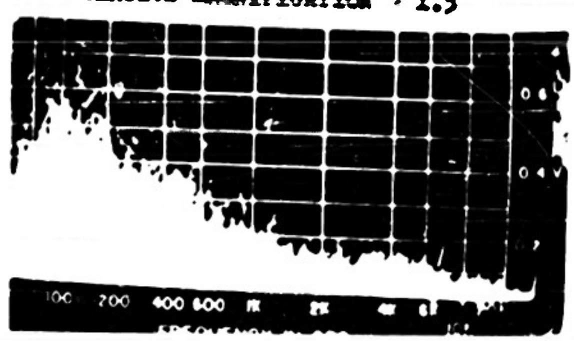
$R_x = 2.87 \times 10^6$
RELATIVE MAGNIFICATION = 1.5



$R_x = 3.07 \times 10^6$
RELATIVE MAGNIFICATION = 1.5



$R_x = 3.37 \times 10^6$
RELATIVE MAGNIFICATION = 1



$R_x = 3.93 \times 10^6$
RELATIVE MAGNIFICATION = 2.0

somewhere near the downstream side of the loop, i.e. at a particular value of R_δ, the frequency present in the boundary layer should be such that the value of $\frac{\beta_r \nu}{U_0^2}$ places the point near the downstream (Branch II) side of the loop. Figure 18 shows the values of $\frac{\beta_r \nu}{U_0^2}$ found in a few of the tests as a function of R_δ. Most of the data is near Branch II of the loop, although a few points fall outside the loop. Some of the error undoubtedly occurred in interpreting the photographs.

It is interesting to note that in some of the tests, the frequency amplified does not change as rapidly as would be predicted. In Figure 17, for example, it is obvious that the amplified frequency remains nearly 1000 cycles for some distance down the plate. In this respect, the experimental results did not agree too well with the results predicted by the theory.

Also of interest in this regard is the frequency which first makes its appearance in the boundary layer as the free stream turbulence is changed. From Figures 15 and 17, it is seen that under conditions of higher turbulence, the frequency which first appears with an amplitude of any importance, i.e. with an amplitude discernable on the photograph of the sonic analyzer screen, is considerably higher than the frequency which first appears under conditions of low free stream turbulence. This is shown in Figure 19, where the value of R_δ at which oscillations first appear is shown as a function of $\frac{u'}{U_0}$ in the free stream. This plot shows clearly that at lower free stream turbulence levels, the waves do not make their appearance until the

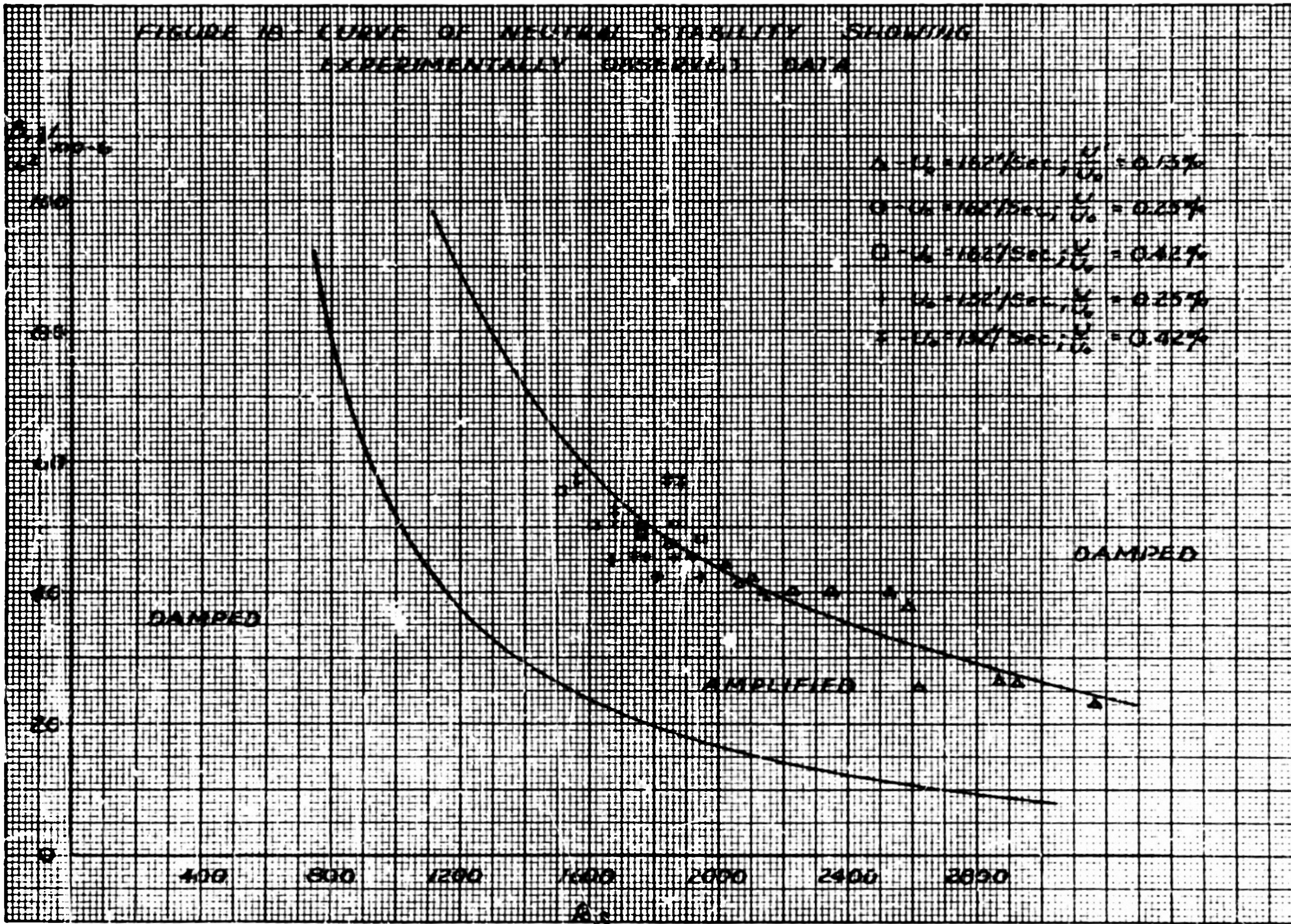
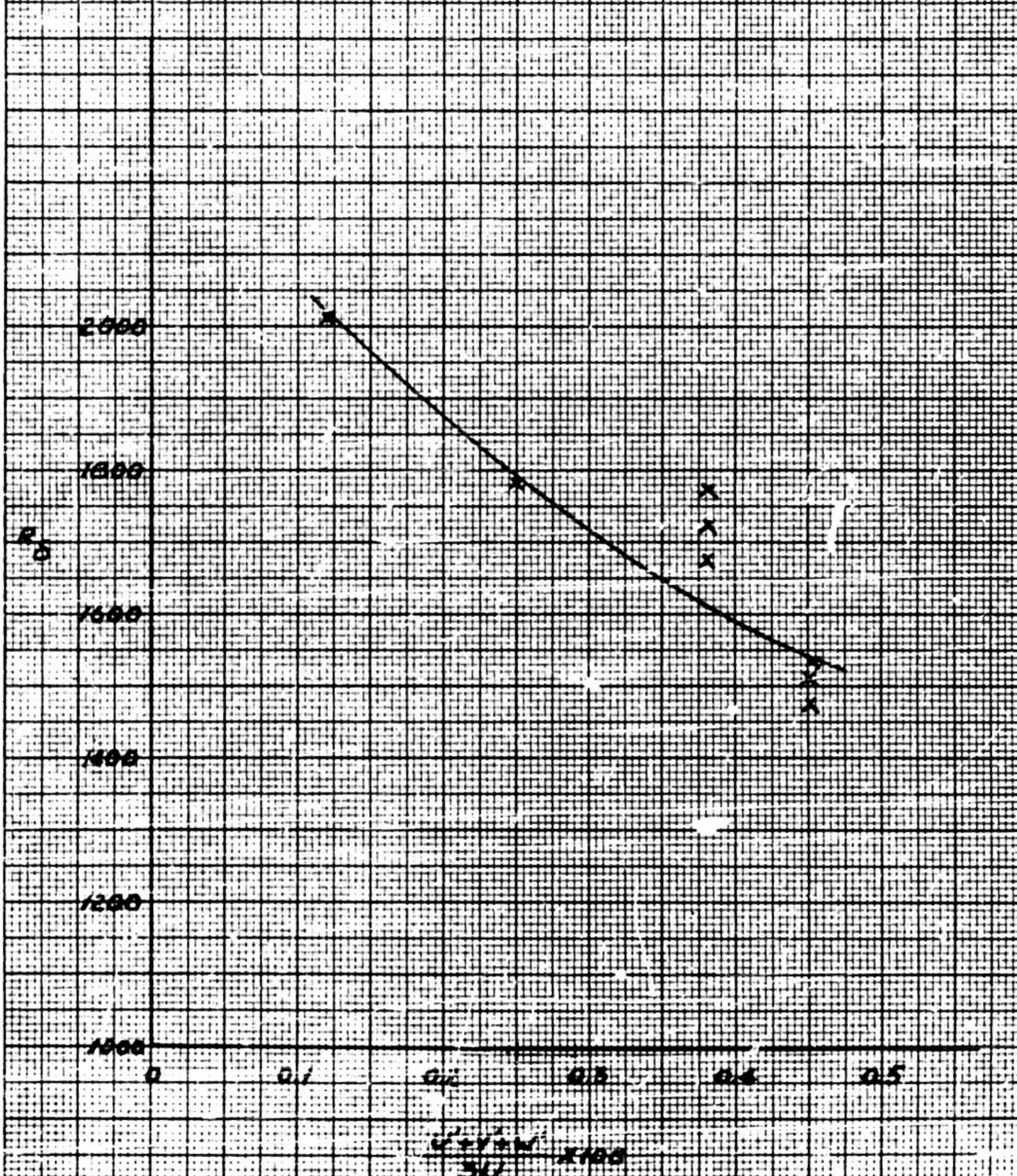


FIGURE 18 - CURVE OF NEUTRAL STABILITY SHOWING EXPERIMENTALLY OBSERVED DATA

FIGURE 19 - VALUE OF R_δ WHEN LAMINAR OSCILLATIONS FIRST APPEAR AS FUNCTION OF FREE STREAM TURBULENCE



observation is made at a higher value of R_δ. This can mean either that there is some minimum value of free stream disturbance necessary to excite a particular frequency, or that the product of disturbance times amplification at the lower turbulence does not give a measurable boundary layer wave i.e. one that would appear above the noise level of the sonic analyzer. This latter conjecture is rather difficult to believe, because, as noted on Figure 1, the amplification given a disturbance increases manyfold with only a slight decrease in $\frac{\beta_r \nu}{U_0^2}$. Assuming that the amplification times the disturbance must reach some value before the wave becomes discernable, it is obvious from Figure 1 that the amplification increases considerably more than the free stream turbulence and that their product is no where near constant, even allowing for some difference in the initial energy distribution with frequency. It almost appears there is some minimum level of free stream disturbance necessary to excite a particular frequency. At any rate, it is an important point to note, that the waves appear at a lower value of R_δ the higher the level of free stream turbulence. Extrapolating this information to considerably higher levels of turbulence, it is apparent that eventually no amplification will be given the disturbances, and at this level of turbulence the laminar oscillations will play no part in transition. This level of turbulence, however, is considerably higher than had heretofore been thought. As noted in an earlier section, the photographs from the panoramic analyzer were analyzed by connecting points of equal optical in-

tensity. If the values are then squared and suitably non-dimensionalized, curves of energy versus frequency result. To non-dimensionalize these curves, the maximum value of the squared output at zero frequency was used. Figure 20 shows the photographs of Figure 15 analyzed in this manner. A certain amount of judgment was necessary in interpreting the photographs, particularly at the lower frequencies where the trace from the panoramic analyzer never seemed to burn into the film with the same intensity as at the higher frequencies, making interpretation somewhat difficult. Also present, particularly at low values of R_x was a certain amount of unexplained low frequency signal, probably due in part to mechanical vibration of the probe. Nevertheless, at higher values of R_x and particularly in the middle range of frequencies (600 to 1500 cycles) the curves of Figure 20 present a true picture of the growth and decay of the laminar oscillations.

It appears that the low frequency end of the spectrum increased rather quickly after the appearance of the waves. This is undoubtedly due to the distortion of the regular frequency as observed in the photograph of Figure 13. As the amplified frequency continues to grow, the low frequency end of the spectrum tends to become constant, and the higher frequencies start to feed off the laminar oscillations. This indicates that the regular wave in the laminar layer cannot undergo further distortion without bursting into random turbulent motion. Also shown for comparison are the spectrum curves for a lower level of free stream turbulence. To be observed in this

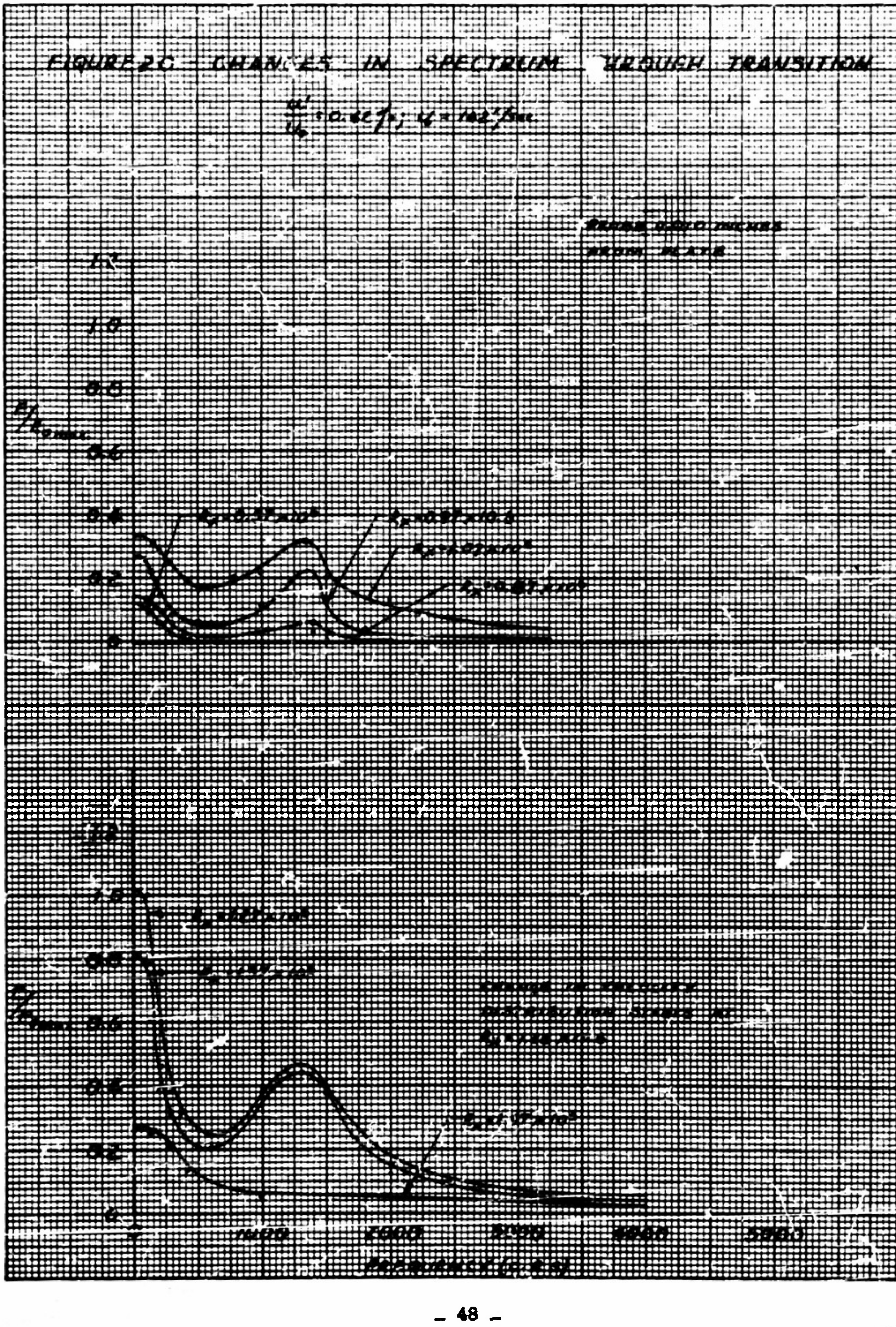


FIGURE 2C CHANGES IN SPECTRUM THROUGH TRANSITION
1.2
1.0
0.8
0.6
0.4
0.2
0
1000
2000
3000
4000
5000

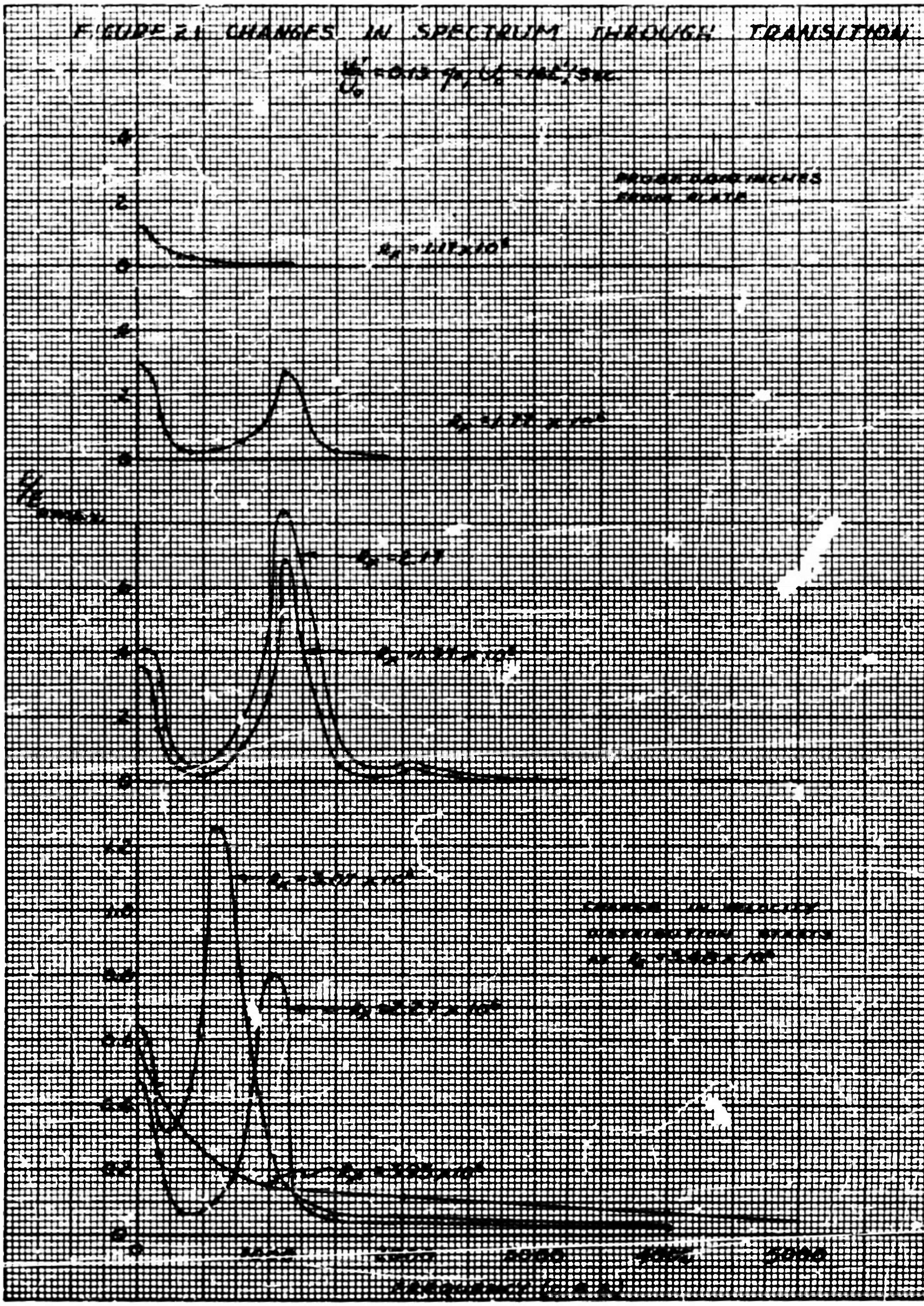


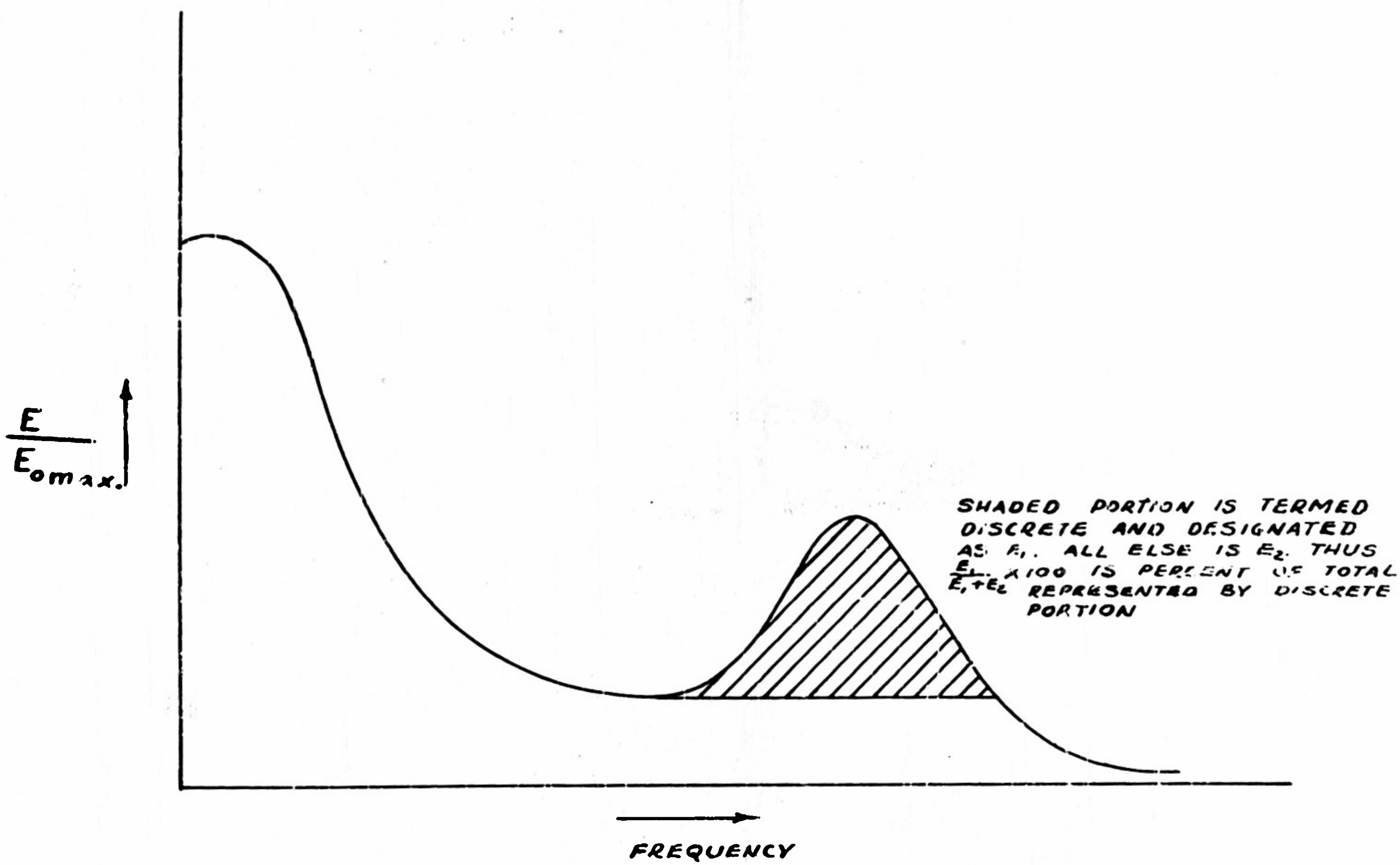
FIGURE 21 CHANGES IN SPECTRUM THROUGH TRANSITION

case is the relative purity of the laminar oscillations. Both with the low turbulence and the high turbulence the curve showing the final spectrum is somewhat misleading with regard to the general level. Once again it must be remembered that all measurements were taken a fixed distance from the surface and in order to get a spectrum measurement without any traces of laminar oscillations it was found necessary to measure some distance following the modification of the laminar velocity distribution. This, of course, meant that the value of $\frac{y}{\delta}$ was considerably less than in previous measurements and caused the decrease in total energy.

An instructive way of showing this data is to separate out from the continuous spectra the energy due to the laminar oscillations. This is very similar to the method used by Roshko (Ref. 10) for presenting his data on the development of turbulent motion from a vortex street in the wake of a cylinder. The situation during transition is remarkably similar to conditions in the immediate wake of a cylinder. In both cases, a large percent of the total energy is, for some distance, concentrated in discrete eddies, which eventually decay into turbulence. One of the essential differences is, of course, that in the case of the laminar layer the total fluctuation energy is increasing, whereas in the case of a cylinder the energy is decaying. Nevertheless, the analogy is a useful one.

Figure 22 shows the method used for separating out the discrete energy caused by the laminar oscillation. It was rather difficult

FIGURE 22 - METHOD USED TO SEPARATE DISCRETE ENERGY FROM CONTINUOUS SPECTRA



to decide upon a suitable method because of the large differences in the curves at the higher frequencies. Admittedly the method used, as indicated in Figure 22, does not show exactly how much of the total energy is represented by the laminar oscillations, but no other method was found that would adequately portray the changes that occur.

It was found very strikingly that the rise and decay of discrete energy was independent of velocity. This is shown in Figure 23, where discrete energy was plotted as a function of R_x for three different speeds. Although there is a certain amount of scatter there seems to be no systematic variation with velocity. This certainly shows the dependence of the entire phenomenon of transition on the free stream turbulence and the Reynolds number. Figure 19 showed that the Reynolds number at which the waves appear is a unique function of the free stream turbulence. Figure 23 indicates that the growth and decay of the discrete energy in the spectrum is a function of Reynolds number, independent of velocity, despite the fact that the frequency found is a function of the reciprocal of the velocity squared.

With the different levels of turbulence used, the rise and decay of discrete energy is shown in Figure 24. For convenience in comparing the curves, experimental points were omitted. As anticipated, with decreasing turbulence, the discrete energy reaches a much greater percent of the total energy before it starts to decrease and with lower turbulence the decrease in discrete energy is much slower than

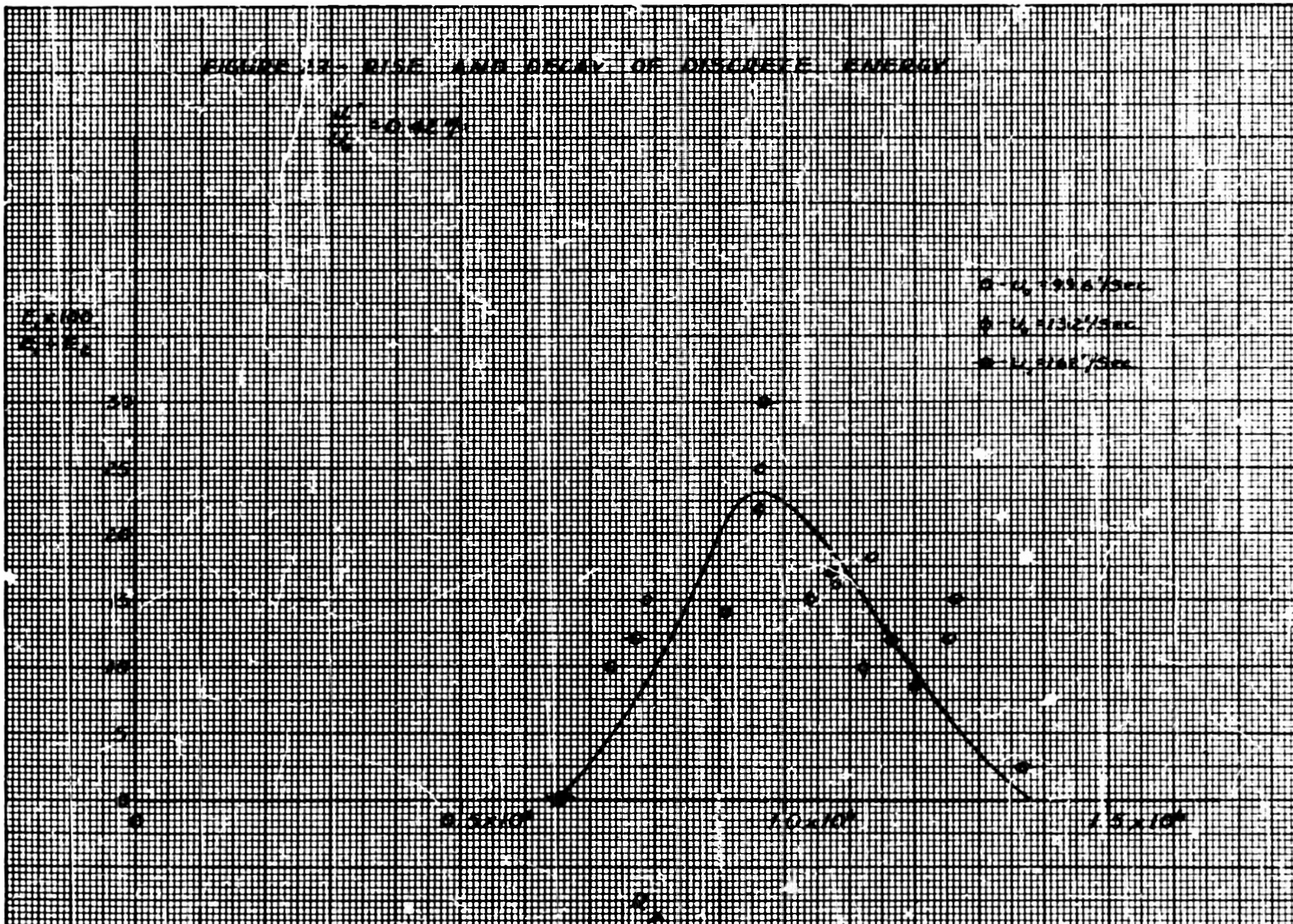
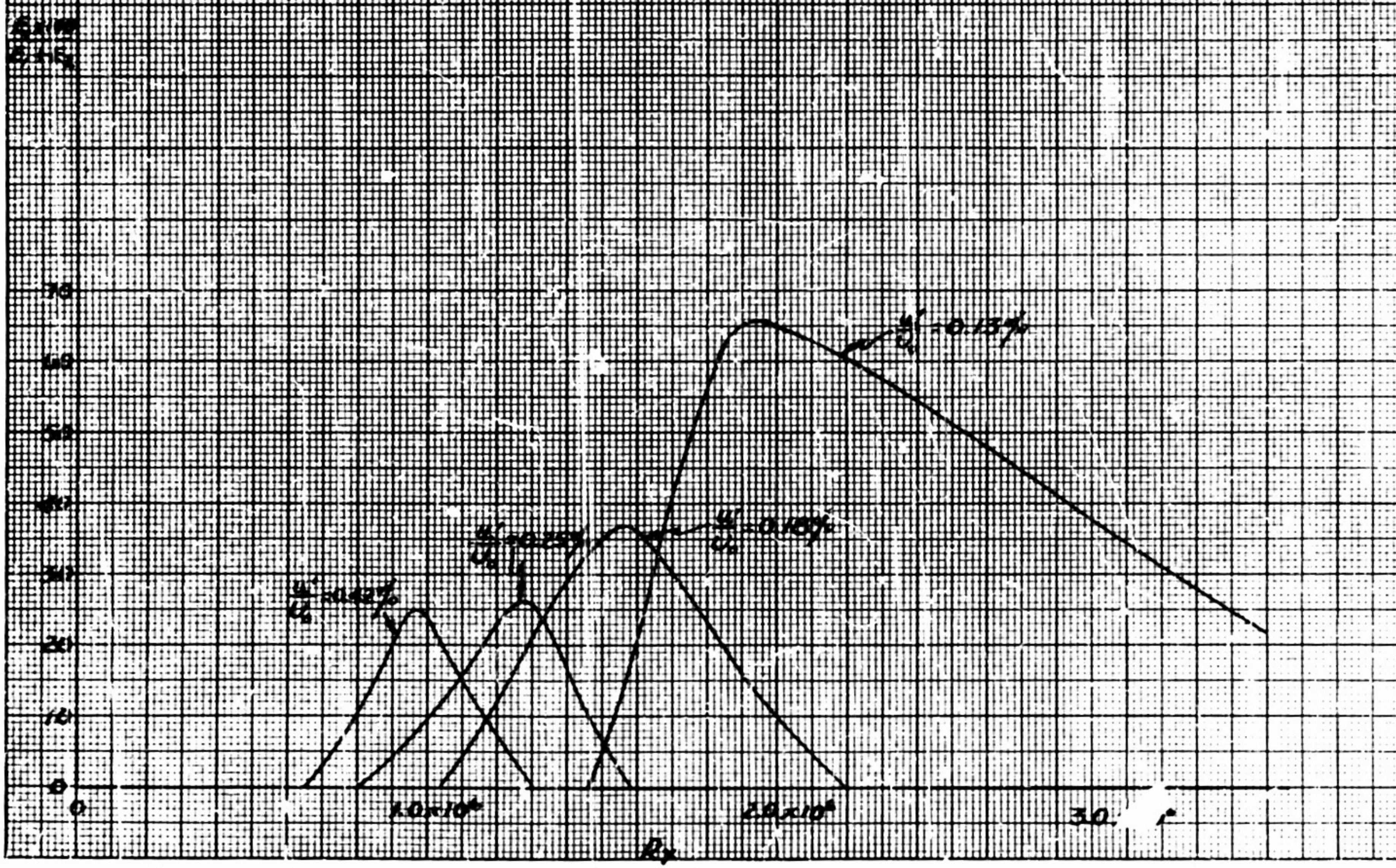


FIGURE 12 – RISE AND DECAY OF DISCRETE ENERGY

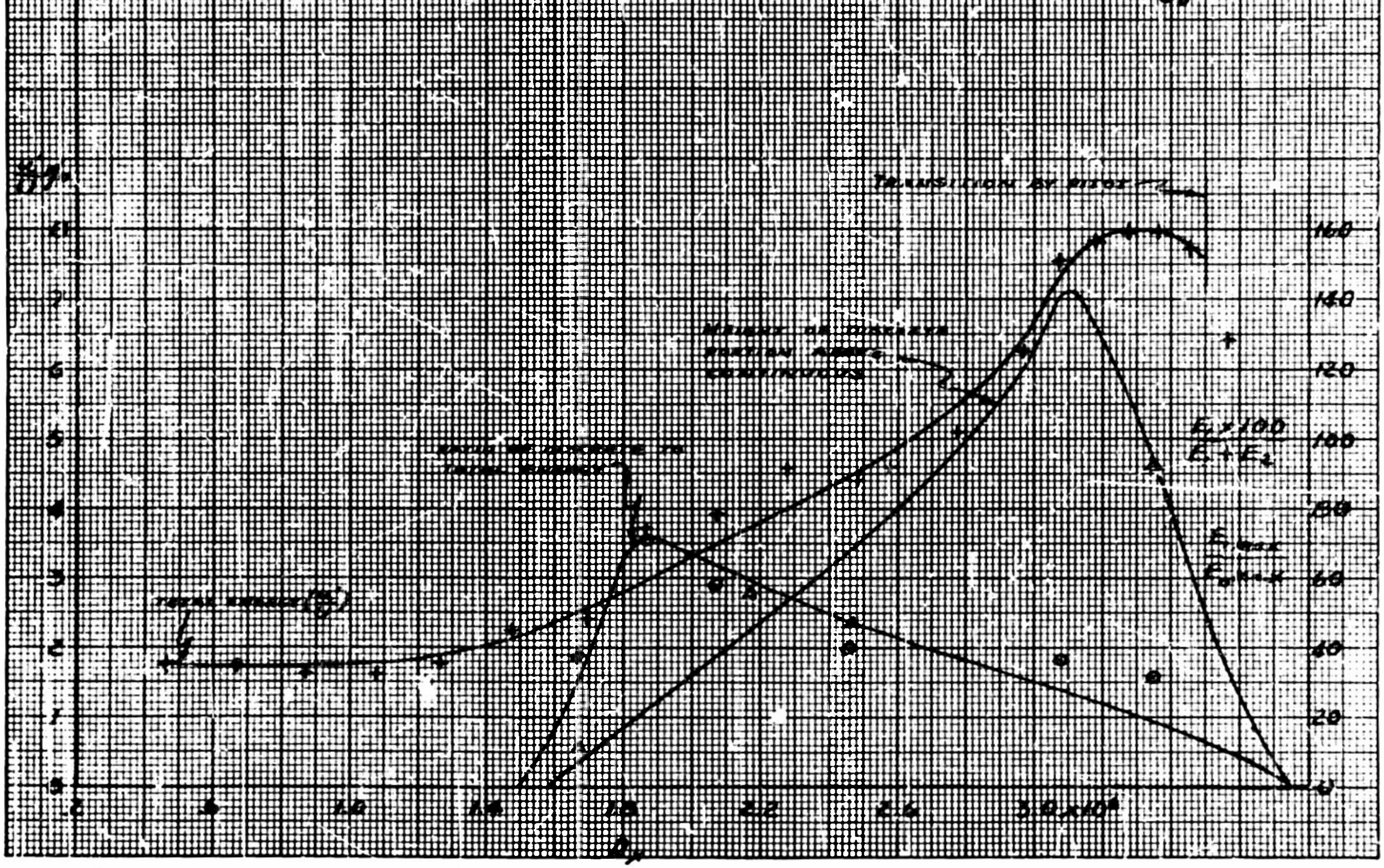
FIGURE 24 RISE AND DECAY OF DISCRETE ENERGY FOR DIFFERENT LEVELS OF FREE STREAM TURBULENCE



with greater degrees of free stream turbulence.

For comparing the differences in transition at different levels of free stream turbulence Figures 25 and 26 tell the story pretty completely. In each case the total energy, the discrete energy and the height of the discrete portion above the continuous is plotted as a function of flat plate Reynolds number. The height of the discrete portion was, of course, non-dimensionalized in the same manner as the curves of Figure 20. As indicated by Figure 25, the total energy starts to increase the same time the discrete energy starts to increase. This corresponds to the initial introduction of the oscillations into the boundary leyer. The discrete energy quickly reaches a peak and starts to decrease, despite the fact that the amplified waves are still present, as indicated by the continuing increase in height of the discrete portion. This indicates that the discrete portion is feeding energy to the rest of the spectrum, initially the lower frequencies and finally the higher frequencies, as discussed above. Eventually the height of the amplified frequency portion of the spectrum starts to decrease, indicating the oscillations are bursting into high frequency turbulence more rapidly than the oscillations can grow. Following this, modification of the velocity distribution occurs quickly, as the pitot tube pressure rise, shown for convenience, indicates. In figure 26, the same curves are shown but for a higher level of turbulence. Aside from transition occurring further forward on the plate, many other differances are discernible. The ratio of

FIGURE 25 - VARIATION OF $\frac{u'}{U_\infty}$, RISE AND DECAY OF DISCRETE ENERGY, AND HEIGHT OF DISCRETE ENERGY PORTION ABOVE CONTINUOUS SPECTRA THROUGH TRANSITION FOR $\frac{u'}{U_\infty} = 0.15\%$



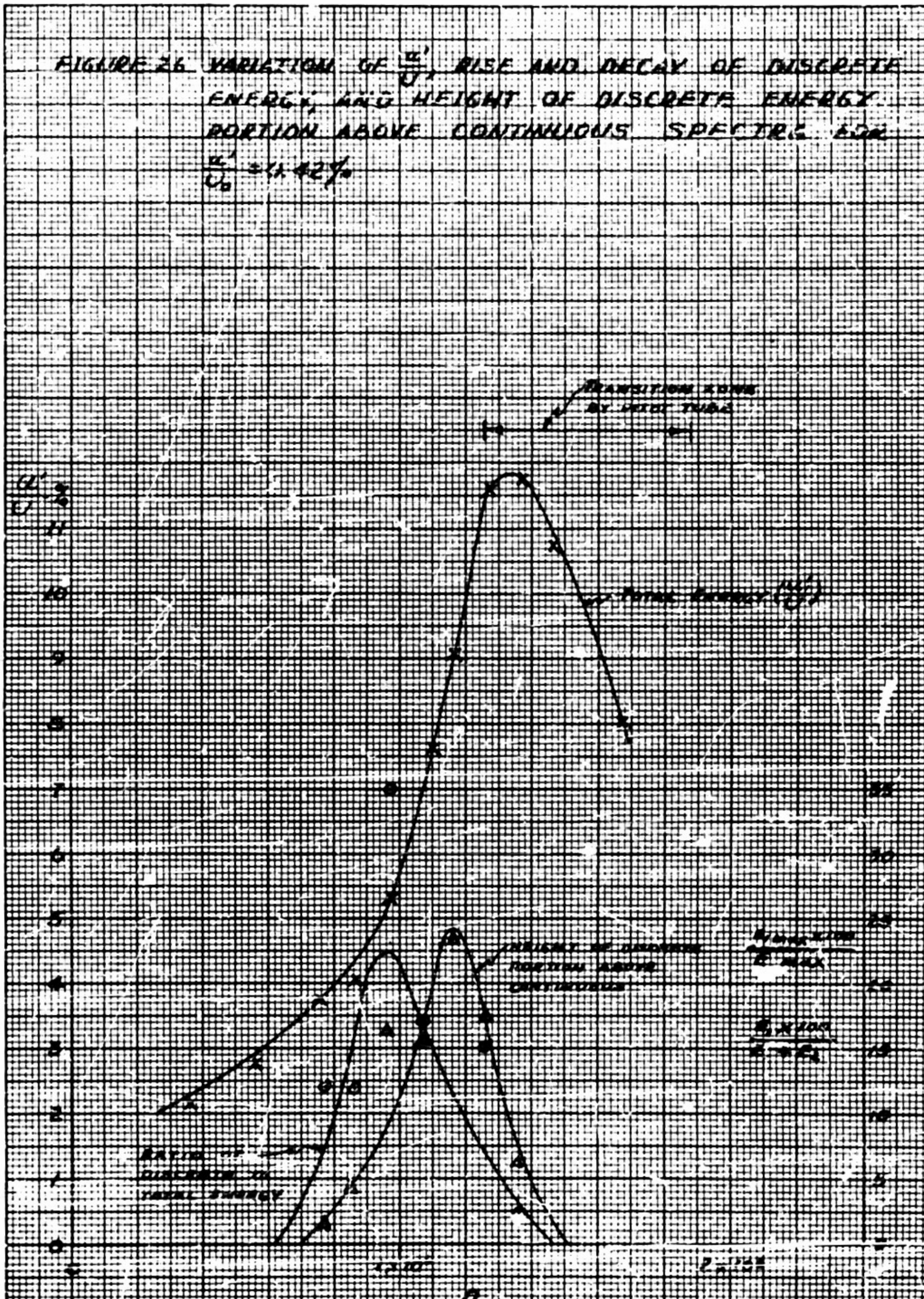


FIGURE 26 VARIATION OF $\frac{u'}{U}$, RISE AND DECAY OF DISCRETE ENERGY, AND HEIGHT OF DISCRETE ENERGY PORTION ABOVE CONTINUOUS SPECTRA FOR $\frac{u'}{U_0} = 1.42\%$

discrete energy to total never becomes very high. This can only mean that as quickly as the oscillations build up, they are distorted and quickly feed the rest of the spectrum. It is seen that the magnitude of the amplified frequency builds up in much the same manner as with lower turbulence, but the maximum is reached much faster and the magnitude of the amplified frequency band relative to the rest of the spectrum decays much more rapidly than with lower turbulence.

As indicated above and borne out by the data, it is very incorrect to speak of transition as occurring at any one point - its beginnings occur very far upstream on the plate. Figure 27 shows the plot of Figure 12 with more of the "landmarks" of transition included. It is seen that with low turbulence, there is considerable distance between the first appearance and the final establishment of the turbulent boundary layer. With higher turbulence, however, the waves are established only a short distance before the turbulent boundary layer washes away the laminar layer.

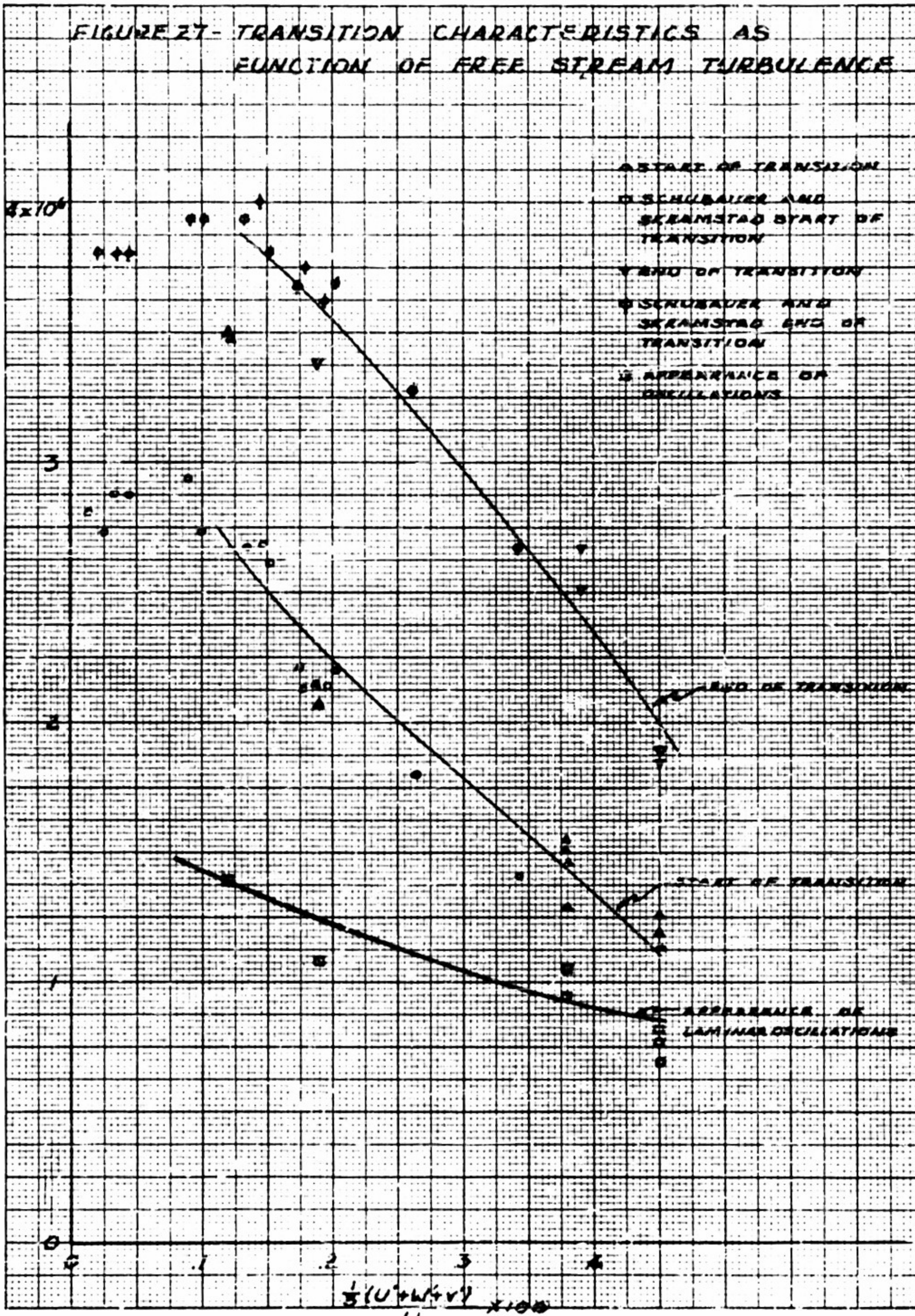


FIGURE 27 - TRANSITION CHARACTERISTICS AS FUNCTION OF FREE STREAM TURBULENCE

CONCLUSIONS

This investigation of the details of the boundary layer flow in the transition region on a smooth flat plate in a zero pressure gradient, indicated the following conclusions:

1. Over the range of turbulence investigated, laminar oscillations play an integral part in transition.
2. The frequency which is amplified at a particular value of boundary layer Reynolds number is determined by the curve of neutral stability resulting from the Tollmien-Schlichting theory of laminar oscillations. The experiments did not indicate always the gradual shifting of the predominate frequency as indicated by the theory.
3. At lower levels of turbulence the oscillations appear at a higher value of boundary layer Reynolds number than at higher levels of free stream turbulence. This indicates either that the product of the initial disturbance and the amplification received at higher values of $\frac{\beta_r \nu}{U_0^2}$ with lower free stream turbulence does not lead to an oscillation of any importance or there is some minimum value of free stream turbulence necessary to excite the higher frequencies.
4. It appears that the mechanism of transition with lower values of free stream turbulence is very similar to the development of a turbulent wake from the vortex sheet behind a cylinder. The oscillations appear at some point determined by the turbulence in the free stream, and quickly build up to an appreciable value.

Shortly after the appearance of the waves the rest of spectrum feeds off the amplified frequency. It appears that first the lower end of the spectrum builds up by virtue of the distortion of the oscillations. Then the higher frequencies gain energy, corresponding to the bursting of the oscillations into turbulence. The amplitude at the lower frequencies remains fairly constant during this process. Shortly after the amplitude at higher frequencies increases, modification of the velocity distribution occurs.

5. Free stream turbulence controls the purity of the laminar oscillations and the rapidity with which the rest of the spectrum feeds off the oscillations. With lower turbulence, a large percent of the total fluctuation energy in the boundary layer in concentrated at the oscillation frequency, and the process covers a considerable distance on the plate. With higher turbulence in the free stream, the rest of the spectrum feeds quickly off of the amplified frequency and only a small portion of the total energy is concentrated at this frequency.

6. Extrapolating this information to even higher levels of free stream turbulence than used for the above tests, it is obvious that at some point the laminar oscillations will play no part in transition. This level of turbulence, however, is much higher than had been previously anticipated. Quite possibly at this level of turbulence and beyond, transition is controlled by the laminar separation theory proposed by Taylor (Ref. 4).

APPLICATION TO MODEL TESTING

The most important application of the above work to ship model test work is the importance of the frequency of the disturbance on the position of transition. In attempting to control the position of transition on a model, it would appear that introducing the proper frequency, possibly with a cylinder of the proper diameter placed a short distance ahead of the model would be worth investigation. It would also appear that trying to predict the position of transition merely from the level of turbulence present in the free stream would be somewhat unreliable in the presence of vibration, etc. A somewhat difficult problem remains in evaluating the characteristics of the turbulent layer after transition has occurred, when the phenomenon is brought about by different means. For example, in the above tests a certain amount of energy was still concentrated at a particular frequency even after the velocity distribution had been modified. This fact alone would make one wonder if the turbulent layer had the same frictional characteristics a given distance following transition irrespective of the degree of free stream turbulence. Certainly the length of the transition region and the energy going into the laminar oscillations particularly when the turbulence level is low would give rise to frictional characteristics which could not be corrected for merely on the basis of present day formulae for resistance with a laminar layer. It would appear that complexity of the transition process as brought out in this report would make a study of the frictional resistance of the transition region a

worthwhile area for effort and such an effort would undoubtedly help to make the corrections for the transition region more reliable.

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